SOUTH DAKOTA MINES
An engineering, science and technology university

CEAD
Computational Engineering Analysis & Design Lab

Advanced Finite Element Methods

ME/BME 736 | Fall 2026

This course develops the mathematical and computational foundations needed to formulate, analyze, and implement advanced finite element methods. Topics include variational formulations, error estimation and adaptive refinement, constrained and mixed methods, nonlinear and transient problems, and PDE-constrained optimization. Heat conduction and solid mechanics provide recurring model problems, and **FEniCSx is used to implement the methods, verify results, and examine numerical behavior**.

COURSE PATH

01 Review & Foundations

Vector and function spaces, Sobolev spaces, integration by parts, variational formulations, energy minimization, heat conduction, linear elasticity, hyperelasticity, finite element approximation, isoparametric mappings, assembly, numerical integration, and boundary conditions.

02 Error Analysis & Adaptivity

A priori error estimation, Galerkin orthogonality, continuity and coercivity, interpolation error, energy and L^2 errors, numerical convergence studies, residual-based a posteriori estimators, local error indicators, marking strategies, and adaptive mesh refinement.

03 Advanced Finite Element Formulations

Constraints using penalty and Lagrange multiplier methods; mixed displacement–pressure formulations for nearly incompressible elasticity; nonlinear residuals, consistent linearization, tangent operators, Newton methods, and hyperelasticity; transient FEM with explicit and implicit time discretization.

04 PDE-Constrained Optimization

Optimization problems governed by PDEs, state and design variables, direct sensitivities, Lagrangian and adjoint formulations, continuous and discrete adjoints, gradient verification, and gradient-based optimization using heat conduction and elasticity examples.

ANALYSIS → FORMULATION → FEniCSx → VERIFICATION → OPTIMIZATION

OPEN COURSE RESOURCES

ceadpx.github.io/finite-element-methods/

github.com/CEADpx/finite-element-methods

Open HTML/PDF book, FEniCSx examples, notebooks, assignments, and data.

FALL 2026

Tue/Thu 9:30–10:45 AM

Classroom Building 206W

Prof. Prashant K. Jha

Prashant.Jha@sdsmt.edu