$$\boxed{\frac{dv}{dt} = g - \frac{c_d}{m} v^2} \quad , \quad 0 < t \le t_F$$

$$v(0) = 0$$

$$(*) \quad \frac{dv}{dt}(t) \approx \frac{v(t+h) - v(t)}{h} \qquad \text{provided } h \text{ is small}$$

$\rightarrow$ $t_0 = 0, \quad t_1 = \Delta t, \quad t_2 = 2\Delta t, \ \ldots \ , \quad t_F = n\,\Delta t$

Δt : time step

$$v_i = v(t_i)$$

from $(*)$

$$\boxed{\frac{dv}{dt}(t_i) \approx \frac{v(t_i + \Delta t) - v(t_i)}{\Delta t}} \quad \text{—} \ (**)$$

from equation of v:

$$\frac{dv}{dt}(t_1) = g - c_d\, v(t_1)^2$$

$$\frac{dv}{dt}(t_2) = g - c_d\, v(t_2)^2$$

$$\vdots$$

$$\frac{dv}{dt}(t_{n-1}) = g - c_d\, v(t_{n-1})^2$$

general i, $\quad i = 1, 2, \ldots, n-1$ $\qquad v_i := v(t_i)$

$$\frac{dv}{dt}(t_i) \approx \frac{v(\overbrace{t_i + \Delta t}^{t_{i+1}}) - v(t_i)}{\Delta t} = g - \frac{c_d}{m} v(t_i)^2$$

$$\Rightarrow \quad \frac{v_{i+1} - v_i}{\Delta t} = g - \frac{c_d}{m} v_i^2$$

$$\Rightarrow \quad \boxed{v_{i+1} = v_i + \Delta t \left(g - \frac{c_d}{m} v_i^2\right)}$$

✓ v_0

✓ $v_1 = v_0 + \Delta t \left(g - \frac{c_d}{m} v_0^2\right)$

✓ $v_2 = v_1 + \Delta t \left(g - \frac{c_d}{m} v_1^2\right)$

$\vdots$

✓ $v_n = v_{n-1} + \Delta t \left(g - \frac{c_d}{m} v_{n-1}^2\right)$

```
for i = 2 : n
    v(i) = v(i-1)
        + Δt (g - cd/m
        v(i-1)^2 )
end
```

Errors in Numerical simulation

× (i) finite capacity of computer in representing numbers

✓ (ii) discretization error, truncation error, numerical error

×, ✓ (iii) modeling error

Box ⇒ "All models are wrong, but some are useful"

×, ✓ (iv) uncertainty / observation error / experimental error in data

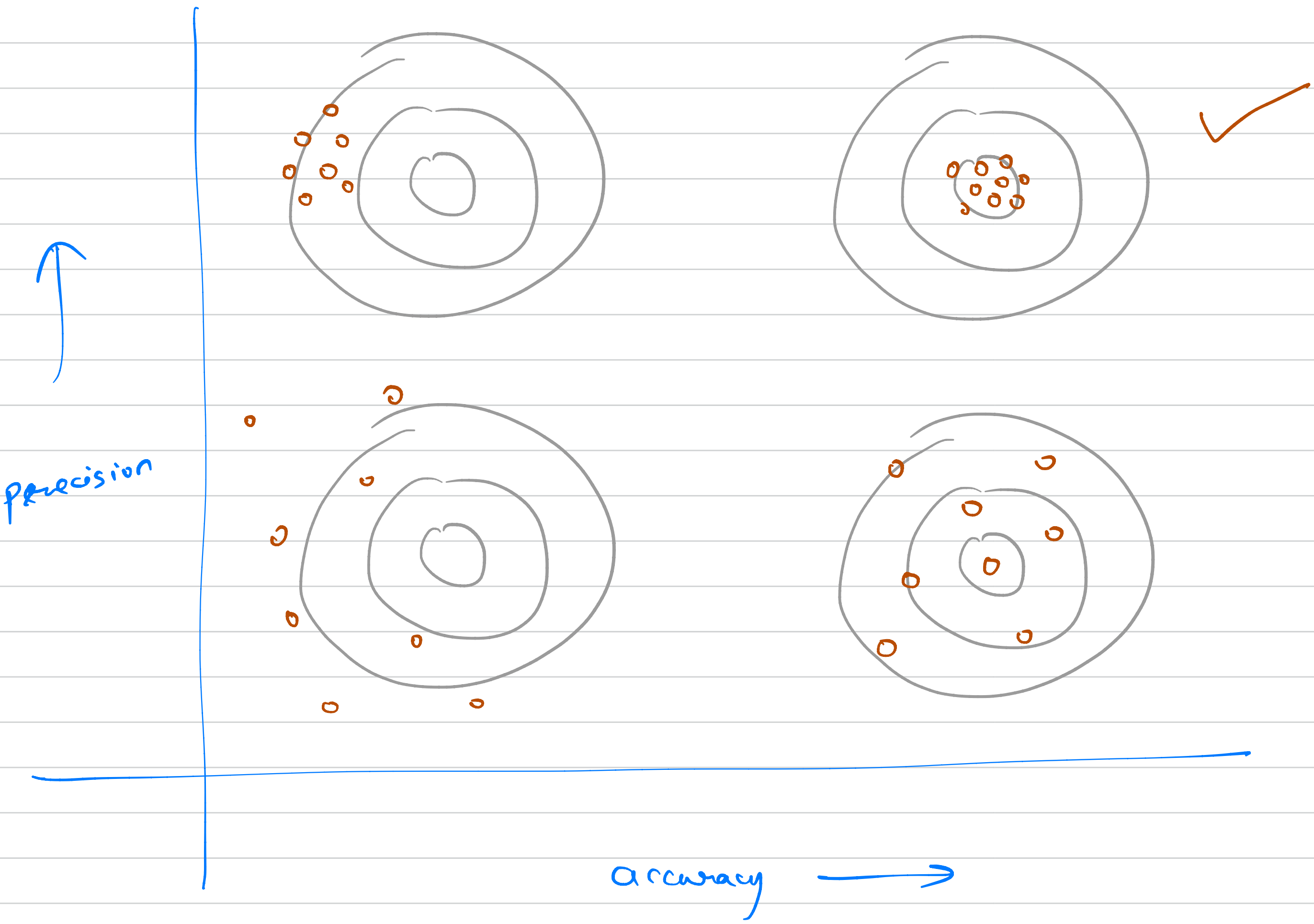
Precision : a_i , $i = 1, \ldots, n$

$$(a_2 - a_1) > (a_3 - a_2) > \ldots > (a_n - a_{n-1})$$

Accuracy : if suppose, I know true value a,

then

$$a - a_i$$



Definition of error :

True error : Applicable only if you know the true value

$$E_t = (\text{True value} - \text{Approximate value})$$

$v_{true} = 10\ m/s$	$v_{true} = 10000\ m/s$
$v_{app} = 9\ m/s$	$v_{approx} = 9999\ m/s$
$E_t = 1\ m/s$	$E_t = 1\ m/s$

$$\boxed{e_t = \frac{E_t}{\text{True value}} \times 100\ \%}$$

$e_t = 1\%$ $\qquad$ $e_t = 0.01\ \%$

$$E_a = \frac{\text{Present approximate value} - \text{Previous approximate value}}{\text{Present approx. val.}} \times 100\%$$