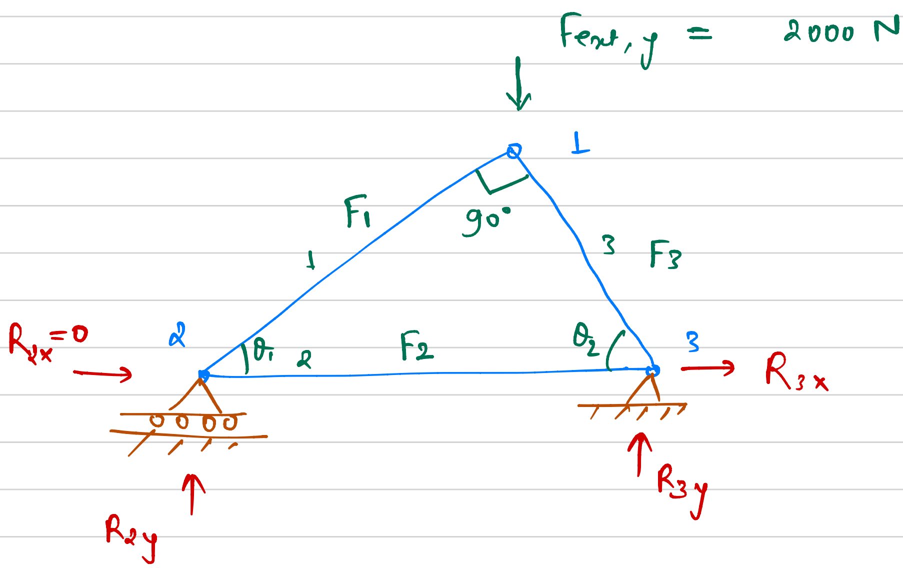
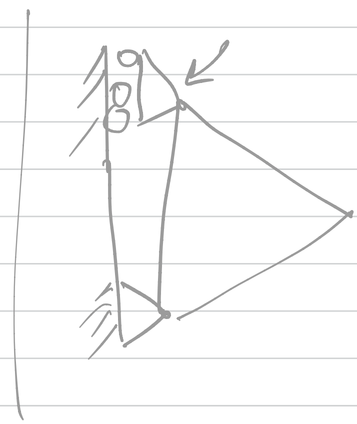
Lecture 11

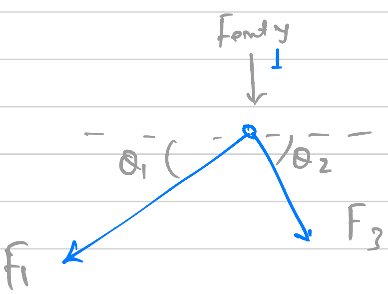
Linear system of equations



Truss structure



Balance of linear momentum

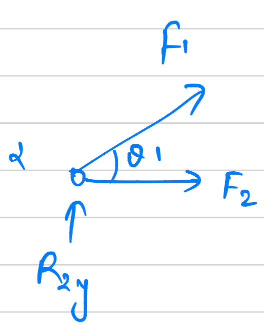


Net horizontal force at 1 $= 0$

(1) $\Rightarrow \quad -F_1 \cos\theta_1 + F_3 \cos\theta_2 = 0$

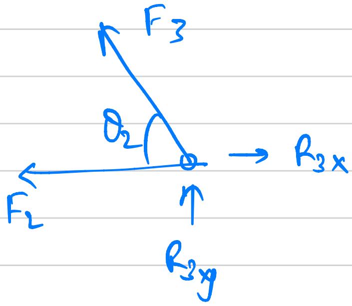
Net vertical force at 1 $= 0$

(2) $\Rightarrow \quad -F_1 \sin\theta_1 - F_3 \sin\theta_2 + F_{ext,y} = 0$



(3) $\quad F_1 \cos\theta_1 + F_2 = 0$

(4) $\quad F_1 \sin\theta_1 + R_{2y} = 0$



(5) $-F_3 \cos\theta_2 - F_2 + R_{3x} = 0$

(6) $\underbrace{F_3}_{a_{63}} \sin\theta_2 + \underbrace{R_{3y}}_{a_{66}=1} = 0$

<u>Unknowns</u>

$F_1, \quad F_2, \quad F_3, \quad R_{2y}, \quad R_{3x}, \quad R_{3y}$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6$

They satisfy equations (1), (2), (3), (4), (5), (6)

and these equations are <u>linear</u> in unknowns

<u>Rewrite in terms of $x_1, x_2, \dots, x_6$</u>

(1) $\Rightarrow$ $a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + a_{14} x_4 + a_{15} x_5 + a_{16} x_6 = b_1$

$a_{11} = -\cos\theta_1, \quad a_{12} = 0, \quad a_{14} = 0, \quad a_{15} = 0, \quad a_{16} = 0, \quad a_{13} = \cos\theta_2, \quad b_1 = 0$

(2) $\Rightarrow$ $a_{21} x_1 + a_{22} x_2 + a_{23} x_3 + \dots + a_{26} x_6 = b_2$

$a_{21} = -\sin\theta_1, \quad a_{22} = 0, \quad a_{23} = -\sin\theta_2, \quad a_{24} = 0, \quad a_{25} = 0, \quad a_{26} = 0$

$b_2 = -F_{ext\,y}$

⋮

(6) $\Rightarrow$ $a_{61} x_1 + a_{62} x_2 + \dots + a_{66} x_6 = b_6$

$$\begin{cases} a_{11} x_1 + a_{12} x_2 + \cdots + a_{16} x_6 = b_1 \\ a_{21} x_1 + a_{22} x_2 + \cdots + a_{26} x_6 = b_2 \\ \vdots \\ a_{61} x_1 + a_{62} x_2 + \cdots + a_{66} x_6 = b_6 \end{cases}$$

$$\Rightarrow \boxed{[a]\{x\} = \{b\}}$$

where

- $\{x\} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_6 \end{bmatrix}$ is a vector of unknown
- $[a] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{16} \\ a_{21} & a_{22} & \cdots & a_{26} \\ \vdots & & & \\ a_{61} & a_{62} & \cdots & a_{66} \end{bmatrix}$ is a matrix of known coefficient
- $\{b\} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_6 \end{bmatrix}$ is a vector of known numbers

Notation : Capital letter for matrix

small letter for column vector

$$\boxed{A x = b}$$

Matrix notation :

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2n} \\ \vdots & & & & & \\ a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{in} \\ \vdots & & & & & \\ a_{m1} & a_{m2} & \cdots & a_{mj} & \cdots & a_{mn} \end{bmatrix} \Rightarrow m \times n \text{ matrix}$$

because

(•) there are m rows

(•) there are n columns

I want element (coefficient) at i^{th} row and j^{th} column

a_{ij} ← this sits at i^{th} row and j^{th} column

Algebra of matrix

- Addition

$A_{m\times n}$, $B_{l\times k}$ ⇒ addition is defined only if

$m = l$

$n = k$

$$C = A + B = \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} & \cdots & a_{1n}+b_{1n} \\ \vdots & & & \\ \vdots & & a_{ij}+b_{ij} & \\ & & & a_{mn}+b_{mn} \end{bmatrix}$$

$$A + B = [a_{ij} + b_{ij}]$$

- Multiplication by a number (scalar)

let α is a number and $A_{m\times n}$ matrix

$$B_{m\times n} = \alpha A_{m\times n} = [\alpha a_{ij}] = \begin{bmatrix} \alpha a_{11} & \alpha a_{12} & \cdots & \alpha a_{1n} \\ & & \alpha a_{ij} & \\ & & & \alpha a_{mn} \end{bmatrix}$$

• <u>multiplication of two matrix</u>

$A_{m\times n}$, $B_{l\times r}$ $\Rightarrow$ multiplication $A\times B$ is defined only if

$n = l$

$\Rightarrow$ number of columns of A

= number of rows of B

multiplication $B\times A$ is defined only if

$r = m$

$\Rightarrow$ number of columns of B

= number of rows of A

$$\underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times 3}}_{A} \underbrace{\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}_{3\times 2}}_{B} = \begin{bmatrix} \cdot\ (1\times1 + 2\times2 + 3\times3) = 14 & \cdot\ (1\times4 + 2\times5 + 3\times6) = 32 \\ \cdot\ ((4,5,6)\times(1,2,3)) & \cdot\ ((4,5,6)\times(4,5,6)) \\ \cdot\ (7,8,9)\times(1,2,3) & \cdot\ (7,8,9)\times(4,5,6) \end{bmatrix}$$

$(a,b,c)\times(e,f,g)$

$= ae + bf + cg$

$$C = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ & & & a_{mn} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1r} \\ b_{21} & b_{22} & \cdots & b_{2r} \\ \vdots & \vdots & & \vdots \\ & & & b_{lr} \end{bmatrix}$$

C_{ij} = [Take ith row of A] × [jth column of B]

$$= [a_{i1}, a_{i2}, \ldots, a_{in}] \times \begin{bmatrix} b_{1j} \\ b_{2j} \\ \vdots \\ b_{lj} \end{bmatrix}$$

$$C_{ij} = a_{i1} b_{1j} + a_{i2} b_{2j} + \cdots + a_{in} b_{lj}$$

$n == l$

$$\boxed{C_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}}$$

$$C_{12} = \sum_{k=1}^{n} a_{1k} b_{k2}$$

Size of $(A \times B)$ = (number of rows in A) × (number of columns of B)

$= m \times r$

Size of $(B \times A) = 1 \times n$