Lecture 40

Linear regression $\quad (x^i, y^i) \quad i = 1, 2, 3, \dots, n$

$$\hat{y} = y(x) = z(x)\, a$$

where $z(x) = [z_1(x), \dots, z_m(x)]$ ← "known" functions

$$a = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix}$$ ← unknown coefficients

errors: squared errors

$$E = E(a) = \frac{1}{n} \sum_{i=1}^{n} \left(y^i - \hat{y}(x^i) \right)^2$$

Problem: find a s.t. $E(a)$ is minimum

Examples: (i) $m = 2$, linear function

$$z(x) = [1, x]$$

(ii) $m = 3$, quadratic function

$$z(x) = [1, x, x^2]$$

(iv) $m = 3$, exponential basis

$$z(x) = [\exp(\lambda_1 x), \exp(\lambda_2 x), \exp(\lambda_3 x)]$$

$\lambda_1, \lambda_2, \lambda_3$ these are fixed numbers

↓

$\lambda_1 \neq \lambda_2 \neq \lambda_3$

(iv) $m = 4$, sinusoidal function

$$z(x) = [\sin(w_1 x), \sin(w_2 x), \sin(w_3 x), \sin(w_4 x)]$$

$w_1 \neq w_2 \neq w_3 \neq w_4$, and w_i are fixed and known

Solving least square problem

finding a that minimize $E(a)$

is equivalent to

Solving $Ja = b$

where

$$b = B^T \begin{bmatrix} y^1 \\ y^2 \\ \vdots \\ y^n \end{bmatrix} \qquad b_{m \times 1}$$

$$J = B^T B \qquad J_{m \times m}$$

$$B = \begin{bmatrix} \text{---} & z(x^1) & \text{---} \\ \text{---} & z(x^2) & \text{---} \\ & & \\ \text{---} & z(x^n) & \text{---} \end{bmatrix}_{n \times m}$$

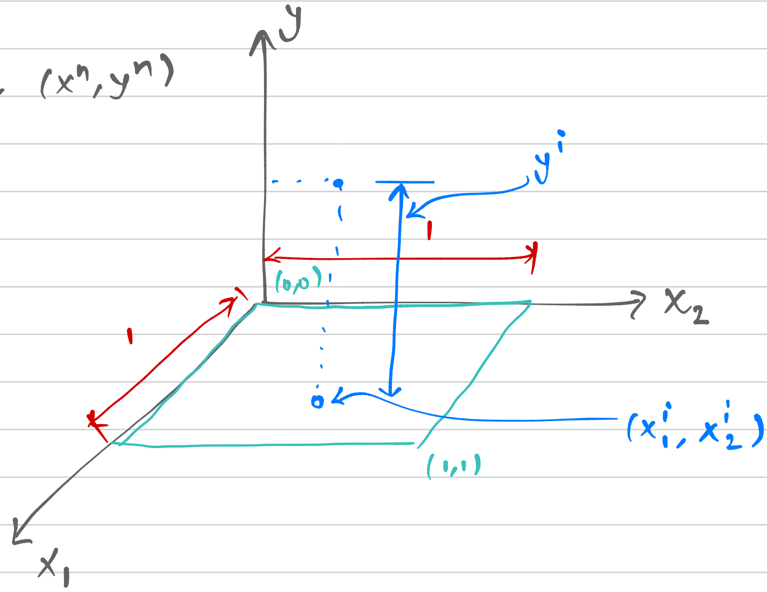
$$= \begin{bmatrix} z_1(x^1) & z_2(x^1) & \cdots & z_m(x^1) \\ z_1(x^2) & z_2(x^2) & \cdots & z_m(x^2) \\ \vdots & & & \\ z_1(x^n) & z_2(x^n) & \cdots & z_m(x^n) \end{bmatrix}$$

linear regression in multiple dimensional

temperature data

$(x^1, y^1), (x^2, y^2), \ldots, (x^n, y^n)$

$$x^i = \begin{bmatrix} x_1^i \\ x_2^i \end{bmatrix}$$



finding model for the data:

$$\hat{y} = \hat{y}(x)$$

$$\uparrow$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Example: (1) Linear regression with linear function

$$\longrightarrow \hat{y}(x) = a_1 + a_2 x_1 + a_3 x_2$$

$$= z(x)\, a$$

$$z(x) = [1, \; x_1, \; x_2], \quad a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

Squared error

$$E = \frac{1}{n} \sum_{i=1}^{n} \left(y^i - \hat{y}(x^i) \right)^2$$

min E with respect to a

$\Updownarrow$ equivalent

$$Ja = b, \qquad b = B^T y = B^T \begin{bmatrix} y^1 \\ y^2 \\ \vdots \\ y^n \end{bmatrix}$$

$$J = B^T B$$

$$B = \begin{bmatrix} \text{---} & z(x^1) & \text{---} \\ \text{---} & z(x^2) & \text{---} \\ & \vdots & \\ \text{---} & z(x^n) & \text{---} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & x_1^1 & x_2^1 \\ 1 & x_1^2 & x_2^2 \\ \vdots & & \\ 1 & x_1^n & x_2^n \end{bmatrix}$$

<u>Example</u> linear regression with linear function in K-dimension

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_K \end{bmatrix}$$

$$\hat{y}(x) = a_1 + a_2 x_1 + a_3 x_2 + \cdots + a_{K+1} x_K$$

$$= z(x)\, a$$

$$z(x) = [1, x_1, x_2, \cdots, x_K]$$