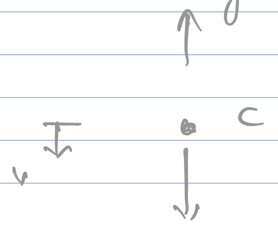
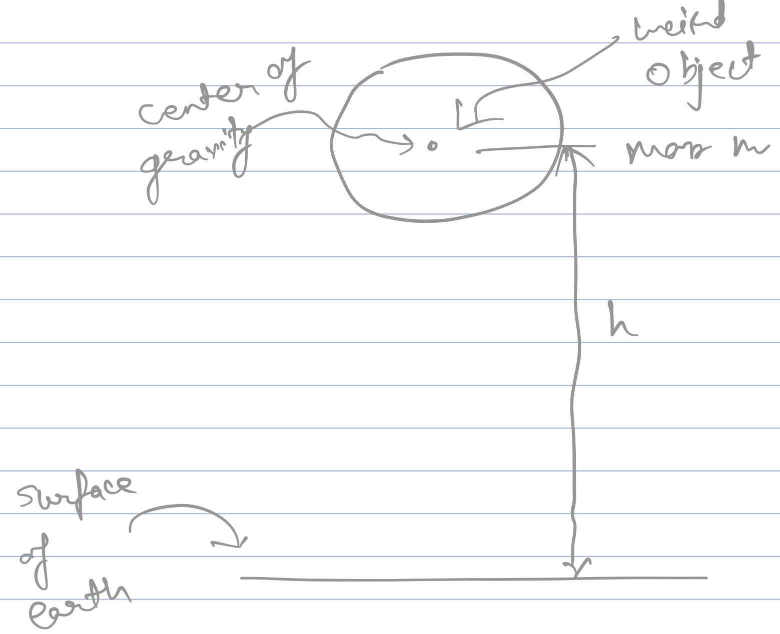
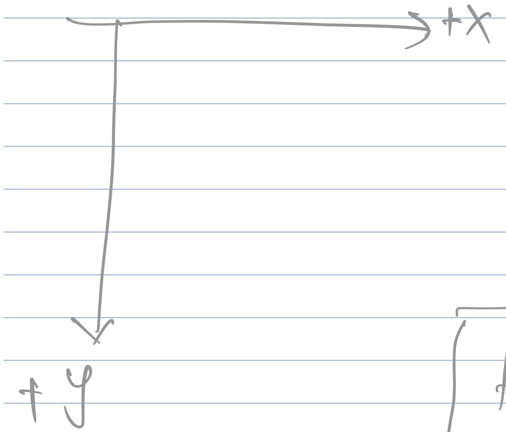
Lecture 2

$F_{drag} : m\, C_d\, |v|\, v$



$F_{gravity} = mg$





$\boxed{mgh : \text{potential}}$

$|v| = \sqrt{v^2}$

$$\boxed{F_{gravity} = mg}$$

$$F_{drag} = m\, C_d\, |v|\, (-v)$$

$$= -m\, C_d\, |v|\, v$$

v is downward $\Rightarrow$ v is always positive

v +ve

$|v| = v$

$$\boxed{F_{drag} = -m\, C_d\, v^2}$$

Drag Coefficient C_d

- density of gas
- shape of an object
- it depends on mass

Conservation law we will use is balance of linear momentum

Rate of change of linear momentum = Internal force + External force

linear momentum = mv

m = constant

$$\frac{d}{dt}(mv) = m\frac{dv}{dt} \qquad \text{LHS}$$

$$\left.\begin{aligned} F_{internal} &= 0 \\ F_{external} &= F_{gravity} + F_{drag} \\ &= mg - mc_d v^2 \end{aligned}\right\} \text{RHS}$$

$$\Rightarrow \quad m\frac{dv}{dt} = mg - \underbrace{mc_d}\, v^2$$

$$C_d = mc_d$$

$$\Rightarrow \quad \boxed{\frac{dv}{dt} = g - \frac{C_d}{m}v^2} \leftarrow \text{Balance of linear momentum}$$

$$v(t=0) = v(0) = \,? = 0$$

$\Downarrow$

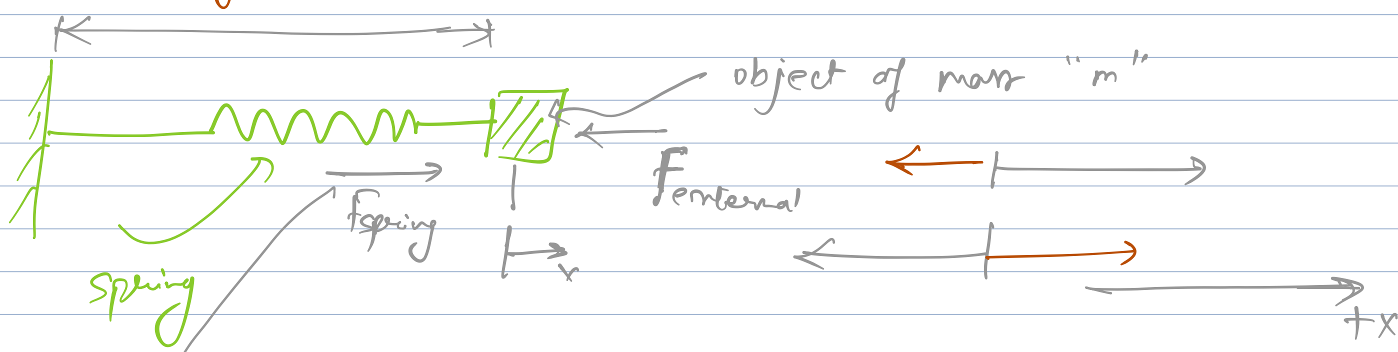
Solve the ordinary differential equation (Initial Value Problem)

$$\boxed{v = \sqrt{\frac{gm}{C_d}} \tanh\left(t\sqrt{\frac{gC_d}{m}}\right)}$$

analytical solution (exact solution)

Other examples of Conservation laws

(I) Spring (Linear momentum balance)



Rate of change of linear momentum

$$= m\frac{dv}{dt}$$

F_{spring} = force generated by spring

$\propto$ change in length of spring

"proportional to"

- Object is at $x=0$ initially.
- Spring is of length l_0 initially

— at any time, so let $x = x(t)$ is a position of object

the length of spring at time t

$$l = l(t)$$

$$\boxed{l(t) = l_0 + x(t)}$$

$$\boxed{\begin{aligned} f &= f(x) \\ &= f(x, y, z) \end{aligned}}$$

change in length of spring at time t

$$\delta l = l(t) - l_0$$

$$= x(t)$$

$$F_{spring} = (-\delta l)\, K$$

Hooke's law (spring constant, stiffness of a spring)

$$\Rightarrow \boxed{F_{spring} = -kx}$$

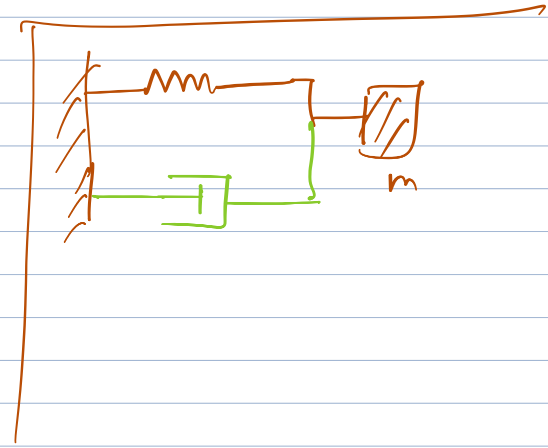
Damping force of spring

↓

"Dampening"

$$F_{damp} \propto v$$

$$= -cv$$



$$m\frac{dv}{dt} = -kx - cv + \text{"}F_{external}\text{"}$$

$$a = \frac{dv}{dt}, \quad \boxed{v = \frac{dx}{dt}}$$

$$= \frac{d^2x}{dt^2}$$

$$\boxed{m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F_{external}}$$

Initial conditions

① position $\quad x(t=0) = x(0) = 0$

② velocity $\quad v = \frac{dx}{dt}(t=0) = 0$

(II) Covid 19 (Conservation of ~~mass~~ "infected" & "deceased" people)

$D = D(t)$ = number of deceased cases in Austin

Rate of change of mass = mass flux

$$\frac{dD(t)}{dt} = \text{growth of deceased cases} - \text{suppression of deceased cases due to policies \& measures}$$

$$= \alpha D^2 - \beta D$$

$$\Rightarrow \boxed{\frac{dD}{dt} = \alpha D^2 - \beta D}$$

$$D(t=0) = M$$