Lecture 16

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

$$\left[\begin{array}{ccc:c} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{array}\right]$$

$\Downarrow$ a_{11} is a pivot

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$f_{21} = \frac{a_{21}}{a_{11}} \rightarrow \quad 0 + a_{22}^{(1)}x_2 + a_{23}^{(1)}x_3 = b_2^{(1)}$$
$$f_{31} = \frac{a_{31}}{a_{11}} \rightarrow \quad 0 + a_{32}^{(1)}x_2 + a_{33}^{(1)}x_3 = b_3^{(1)}$$

$$\left[\begin{array}{ccc:c} a_{11} & a_{12} & a_{13} & b_1 \\ 0 & a_{22}^{(1)} & a_{23}^{(1)} & b_2^{(1)} \\ 0 & a_{32}^{(1)} & a_{33}^{(1)} & b_3^{(1)} \end{array}\right]$$

$\Downarrow$

$$\left[\begin{array}{ccc:c} a_{11} & a_{12} & a_{13} & b_1 \\ 0 & a_{22}^{(1)} & a_{23}^{(1)} & b_2^{(1)} \\ 0 & 0 & a_{33}^{(2)} & b_3^{(2)} \end{array}\right]$$

$a_{22}^{(1)}$ is also a pivot

$$\frac{a_{32}^{(1)}}{a_{22}^{(1)}} = f_{32}$$

$a_{33}^{(2)}$ is also a pivot

$\Downarrow$

Different types of matrices

- Square matrix : # rows = # columns

Transpose of a matrix :

$$A_{m \times n} = [a_{ij}]$$

$$B_{n \times m} = [a_{ji}]$$

$$\underbrace{\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}}_{A} \Rightarrow \underbrace{\begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & & a_{m2} \\ \vdots & \vdots & & \vdots \\ a_{1n} & a_{2n} & & a_{mn} \end{bmatrix}}_{B}$$

$$\boxed{b_{ij} = a_{ji}}$$

$$b_{12} = a_{21}$$

$$b_{n1} = a_{1n}$$

$$A_{m\times n} \longrightarrow B_{n\times m} \quad \Big| \Rightarrow A^T = B$$

- Symmetric matrix — square matrix A which satisfies

$$A^T = A$$