Lecture 15

$$
\begin{aligned}
&(1)\quad a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1\\
&(2)\quad a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2\\
&(3)\quad a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3
\end{aligned}
\qquad
A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix},\ b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}
$$

$\Downarrow$

$$
\begin{aligned}
&(1)\quad a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2\\
&(2)\quad a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1\\
&(3)\quad a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3
\end{aligned}
\qquad
A = \begin{bmatrix} a_{21} & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}
$$

$\Downarrow$

$x_2 \longrightarrow x_1$, $x_1 \longrightarrow x_2$

$$
\begin{aligned}
a_{22}x_1 + a_{21}x_2 + a_{23}x_3 &= b_2\\
a_{12}x_1 + a_{11}x_2 + a_{13}x_3 &= b_1\\
a_{32}x_1 + a_{31}x_1 + a_{33}x_3 &= b_3
\end{aligned}
$$

$$
\begin{bmatrix} a_{22} & a_{21} & a_{23} \\ a_{12} & a_{11} & a_{13} \\ a_{32} & a_{31} & a_{33} \end{bmatrix}
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}
=
\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}
$$

$Ax = b$ $\qquad x,\ y$

$\Rightarrow \quad Ax = b \ , \quad Ay = b$

$\Rightarrow \quad A(x-y) = 0$

If I have A^{-1}: $\qquad A^{-1}A(x-y) = A^{-1}0 = 0$

$$\Rightarrow \quad x - y = 0$$

$$\Rightarrow \quad x = y$$

for A to have A^{-1}, $\det A \neq 0$

$$\left.\begin{aligned} a_{11} x_1 + a_{12} x_2 + a_{13} x_3 &= b_1 \\ a_{21} x_1 + a_{22} x_2 + a_{23} x_3 &= b_2 \\ a_{31} x_1 + a_{32} x_2 + a_{33} x_3 &= b_3 \end{aligned}\right\} \quad a_{11} = 0$$

$$\left.\begin{aligned} a_{11} x_1 + a_{12} x_2 + a_{13} x_3 &= b_1 \\ a_{21} x_1 + a_{22} x_2 + a_{23} x_3 &= b_2 \\ a_{31} x_1 + a_{32} x_2 + a_{33} x_3 &= b_3 \end{aligned}\right\} \quad a_{22} - \frac{a_{12}}{a_{11}} = 0$$