Lecture 12

Recap:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

i^{th} row, j^{th} column a_{ij}

- <u>addition</u> $A_{m\times n}$, $B_{m\times n}$

$$C_{m\times n} = A + B \Rightarrow [c_{ij}] = [a_{ij} + b_{ij}]$$

- <u>Scalar multiplication</u>: α number, $A_{m\times n}$

$$C_{m\times n} = \alpha A_{m\times n} \Rightarrow [c_{ij}] = [\alpha a_{ij}]$$

- <u>Subtraction</u>

$$C_{m\times n} = A - B \Rightarrow [c_{ij}] = [a_{ij} - b_{ij}]$$

- <u>Multiplication of two matrices</u>

$A_{m\times n}$, $B_{l\times r}$

① $A \times B$ works only if # columns of A = # rows of B

$\Rightarrow n = l$

$C = A \times B \Rightarrow C$ is $m \times r$ matrix

$$[c_{ij}] = [\sum_{k=1}^{n} a_{ik} \, b_{kj}]$$

(2) $B \times A$ works only if # columns of B = # rows of A

$\Rightarrow p = m$

$$D = B \times A \Rightarrow [d_{ij}] = [\sum_{k=1}^{p} b_{ik} \, a_{kj}]$$

- Multiplication of matrix and vector :

(i) think of row vector as $1 \times n$

think of column vector as $n \times 1$

▫ Then $\underbrace{A_{m \times n} \, X}$ works only for column vector if $n > 1$

$\downarrow$

$X_{n \times 1}$

$$A_{m \times n} X_{n \times 1} = y_{m \times 1} \leftarrow \text{column vector}$$

$$y = AX \Rightarrow [y_{ij}] = [\sum_{k=1}^{n} a_{ik} \, x_{kj}] \qquad j = 1$$

$y_{ij} \longrightarrow y_i$, $x_{k\cdot} \longrightarrow x_k$

$$\boxed{y_i = \sum_{k=1}^{n} a_{ik} x_k}$$

If I have A and B matrices, x, y column vectors, &

$\alpha > 0$ number

- $(A+B)x = Ax + Bx$
- $\alpha(A+B)x = \alpha Ax + \alpha Bx$
- $A(x+y) = Ax + Ay$

~~$\alpha 4 = 1$~~

$\Rightarrow \alpha\beta = 1$

$\Rightarrow \beta = \alpha^{-1}$

$$A^{-1}(Ax) = x$$

$$A(A^{-1}x) = x$$

$$Ax = b \Rightarrow \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{16} \\ a_{21} & a_{22} & \cdots & a_{26} \\ \vdots & & & \\ a_{61} & a_{62} & \cdots & a_{66} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_6 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_6 \end{bmatrix}$$

If there is a inverse of A:

$A_{n \times n}$

$$Ax = b$$

$$y = Ax$$

$$A^{-1}y = A^{-1}(Ax)$$

$$= x$$

$$\Rightarrow \boxed{A^{-1}y = x}$$

$$A^{-1}b = \bar{b}$$

Inverse is defined as:
B is called inverse of A if and only if

$$\boxed{BAx = ABx = x}$$

for any x column vector

(·) $A_{n \times n}$ ← Square matrix

(·) $x_{n \times 1}$

(·) $B_{n \times n}$

A^{-1}

$\Rightarrow \quad A^{-1}(Ax) = A^{-1}b$

$\Rightarrow \quad \boxed{x = A^{-1}b = \bar{b}}$

Methods to solve $Ax=b$

- direct method
  - inverse
  - Cramer's rule
- Graphical method
- Numerical methods
  - (◦) Iterative method
  - (◦) partial pivoting method

$Ax=b \Rightarrow \begin{matrix} a_{11}x_1 + \cdots + a_{1n}x_n = b_1 \\ \vdots \\ a_{n1}x_1 + \cdots + a_{nn}x_n = b_n \end{matrix}$

From here onwards: $A_{n\times n}$

- size of matrix $= n$
- size of the system $=$ # of equations $=$ # rows of A

Direct (inverse) method:

$\underset{2\times 2}{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$B_{2\times 2} \Rightarrow = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$

$BAx = ABx$

$AB = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{21} \\ a_{21}b_{11} + a_{22}b_{12} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$

$C = BA = \begin{bmatrix} & & \end{bmatrix}$

$D = AB$

$BAx = x \Rightarrow$

$$\boxed{\begin{aligned} c_{11} x_1 + c_{12} x_2 &= x_1 \\ c_{21} x_1 + c_{22} x_2 &= x_2 \\ \\ d_{11} x_1 + d_{12} x_2 &= x_1 \\ d_{21} x_1 + d_{22} x_2 &= x_2 \end{aligned}}$$

$ABx = x \Rightarrow$

for all $\underline{x}$

x_1 & x_2

$x_1 = 1, \ x_2 = 0$

$x_1 = 0, \ x_2 = 1$

$$A_{1\times1} = [a_{11}] \Rightarrow \det A = a_{11}$$

$$A_{2\times2} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \Rightarrow \det A = a_{11} a_{22} - a_{21} a_{12}$$

$$A_{3\times3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \Rightarrow \det A = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$A^{-1} = \frac{1}{\det A} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} \Rightarrow \begin{aligned} AA^{-1}x &= x \\ A^{-1}Ax &= x \end{aligned}$$

MATLAB

$$\boxed{x = A^{-1} b} \rightarrow \boxed{\begin{aligned} x &= A \backslash b \\ &= \text{inv}(A) * b \end{aligned}}$$

- Graphical method

System of two linear equations

$$a_{11} x_1 + a_{12} x_2 = b_1$$

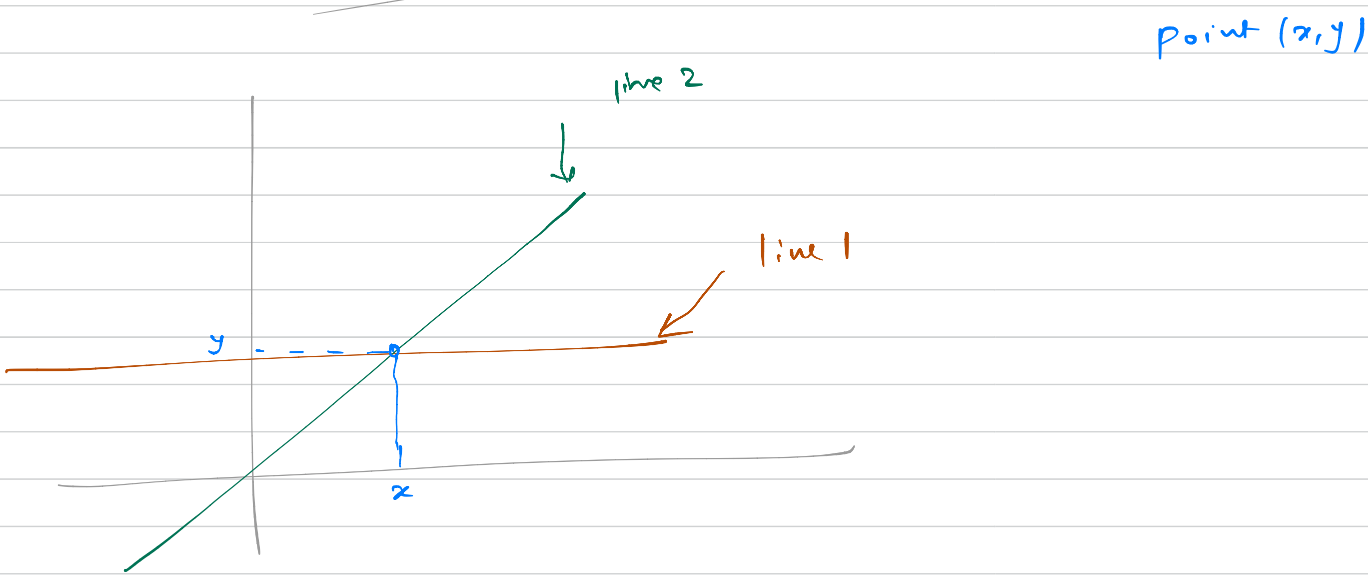
$$a_{21} x_1 + a_{22} x_2 = b_2$$

$x_1 \rightarrow x$

$x_2 \rightarrow y$

$$\boxed{y = \frac{b_1}{a_{12}} - \frac{a_{11}}{a_{12}} x}$$

$$\boxed{y = \frac{b_2}{a_{22}} - \frac{a_{21}}{a_{22}} x}$$



System 1

$$-\frac{1}{2} x_1 + x_2 = 1$$

$$-\frac{1}{2} x_1 + x_2 = \frac{1}{2}$$

Cramer's Rule

$$a_{11} x_1 + a_{12} x_2 = b_1$$