Lecture 5

- ✓ Function files (how these are different from script files)
- Conditional statements, for loop, while loop
- Numerical solution of Free-Fall gravity problem
- Error in numerical solution

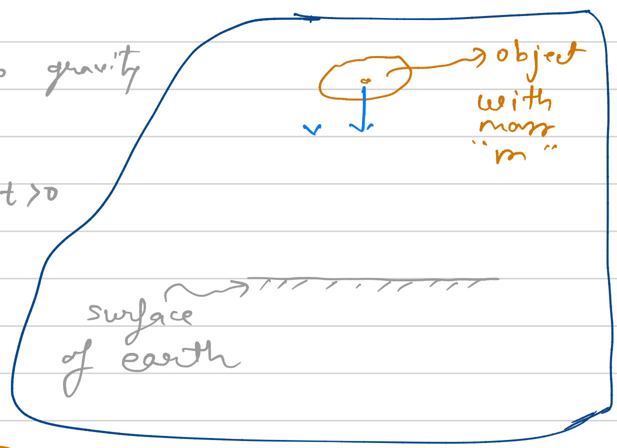
Function file — Create new function that takes in the input (scalar/vector/matrix, one or many inputs) and performs computation and produces output (scalar/vector/matrix, one or many outputs)

Recall Velocity of object falling due to gravity satisfies

$$\boxed{\begin{aligned}\frac{dv(t)}{dt} &= g - \frac{c_d}{m} v^2 \\ v(0) &= 0\end{aligned}} \rightsquigarrow \text{for any } t,\ t>0$$

$\Downarrow$ Exact solution

$$\boxed{v(t) = \sqrt{\frac{gm}{c_d}}\,\tanh\left(t\sqrt{\frac{g c_d}{m}}\right)}$$



We create a MATLAB function that computes velocity of an object at specified time and for specified drag coefficient

~~gravity Example.m~~ freefall.m

```
function v = freefall(t, Cd)
% freefall: compute velocity of free falling object assuming mass
            m = 1 kg
% v = freefall(t, Cd)
% input:
%     t = time (s)   vector of time   (scalar or vector)
%     Cd = drag coefficient (kg/m)   (scalar)
% Output:
%     v = downward velocity (m/s)   (scalar or vector)
g = 9.81;      % gravity acceleration
m = 1;
a = sqrt(m*g/Cd);
b = sqrt(g*Cd/m);
v = a*tanh(b*t);
```

- Conditional statements in MATLAB

(1) IF ELSE

```
if  condition        x >= 0
      statement (task)
end
```

x ?

sqrt(x) → x < 0

sqrt(x) → x ≥ 0

```
if  condition                    → x ≥ 0
        statement 1              → disp ( sqrt(x) )
else
        statement 2              → disp ( sqrt (-x) )
end
```

$n = 1, 2, 3, \ldots, 5$

```
if  condition 1                  → "n=1"
        statement 1
elseif  condition 2              → "n=2"
        statement 2
   :
else                             → n ≠ 1, n ≠ 2, ..., n ≠ 5
        statement N
end
```

2. SWITCH

```
switch    test expression

      case    value 1

              statement 1

      case    value 2

              statement 2
       :
      otherwise
              statement N
end
```

```
switch   n

      case   n = 1
               do task 1
      case   n = 2
               :

      otherwise
               do task 6
end
```

- Loops

1. FOR LOOP

```
for index = start : step : finish
    task that may or may not depend
    on index
end
```

```
for n = 1:1:5
    disp(n)
end
```

```
while True
    statement
end
```

2. WHILE LOOP

```
while condition
    statement
end
```

} stops when condition is no longer true.

```
n = 1;
while "n ≤ 5"
    disp(n)
    n = n+1
end
```

- **Numerical solution** Suppose exact solution for the problem

$$\frac{dv}{dt} = g - \frac{c_d}{m} v^2$$ with $v(0) = 0$ is not known.

$v(0)$ non zero value

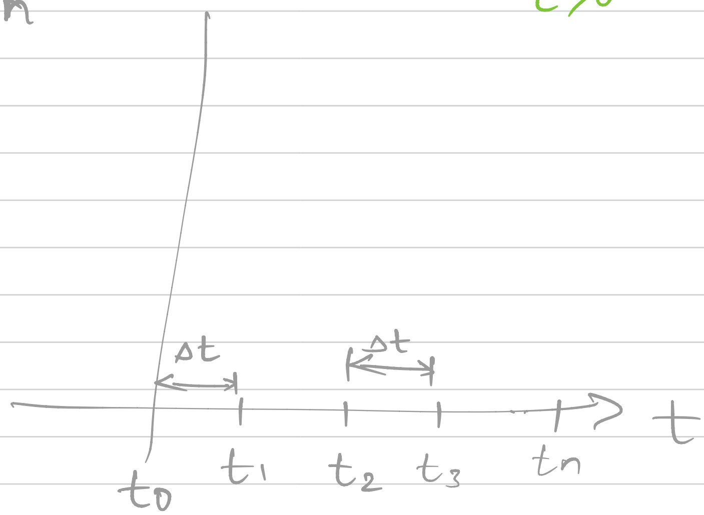
How do I still solve the problem?

"Numerical method"

$$\frac{dv}{dt}(t) = g - \frac{c_d}{m} v(t)^2$$

$t > 0$

$t_0 = 0,\ t_1,\ t_2, \ldots,\ t_n$



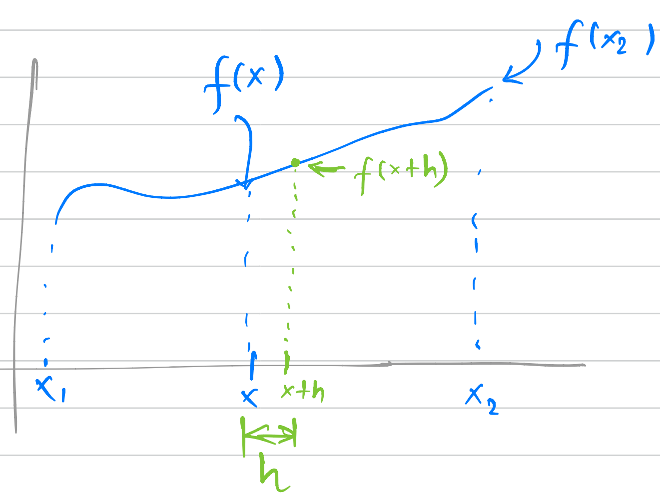
$$(*)\ \begin{cases} \dfrac{dv}{dt}(t_1) = g - \dfrac{c_d}{m} v(t_1)^2 \\ \dfrac{dv}{dt}(t_2) = g - \dfrac{c_d}{m} v(t_2)^2 \\ \vdots \\ \dfrac{dv}{dt}(t_n) = g - \dfrac{c_d}{m} v(t_n)^2 \end{cases}$$

$f = f(x)$, x is a variable $x_1 \le x \le x_2$

Derivative of function

$$\frac{df}{dx}(x) = \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{(x+h) - x} \right)$$

$$\boxed{\frac{df}{dx}(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$



$$\frac{df}{dx}(x) \approx \frac{f(x+h) - f(x)}{h}$$

provided h is small

$$\frac{dv}{dt}(t_1) \approx \frac{v(t_1 + \Delta t) - v(t_1)}{\Delta t} = \frac{v(t_2) - v(t_1)}{\Delta t}$$

in general

$$\frac{dv}{dt}(t_i) \approx \frac{v(t_{i+1}) - v(t_i)}{\Delta t}$$

define

$$v_i := v(t_i)$$

$$\frac{dv}{dt}(t_i) \approx \frac{v_{i+1} - v_i}{\Delta t}$$

Substitute to discrete set of equations (*)

$V(0) =$

$i = 0$

$$\boxed{\frac{v_1 - v_0}{\Delta t} = g - \frac{C_d}{m} v_0^2}$$

$$\Rightarrow \boxed{v_1 = \Delta t \left(g - \frac{C_d}{m} v_0^2\right) + v_0}$$

v_i

$i = 1$

$$\frac{V_2 - V_1}{\Delta t} = g - \frac{C_d}{m} V_1^2$$

$$\Rightarrow \boxed{V_2 = \Delta t \left(g - \frac{C_d}{m} V_1^2\right) + V_1}$$

.

.

.

$i = n-1$

$$\frac{V_n - V_{n-1}}{\Delta t} = g - \frac{C_d}{m} V_{n-1}^2$$

$$\Rightarrow \boxed{V_n = \Delta t \left(g - \frac{C_d}{m} V_{n-1}^2\right) + V_{n-1}}$$