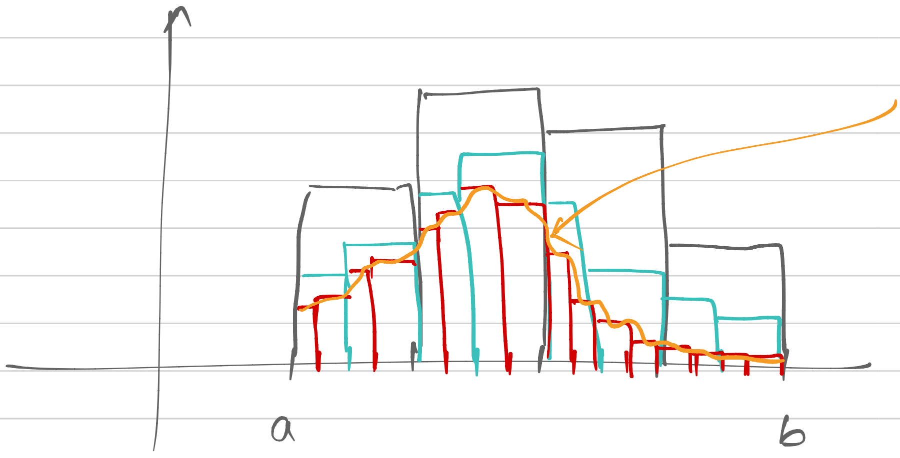
Lecture 25



probability distribution function (pdf)

pdf $\quad f : [a, b] \to [0, 1]$

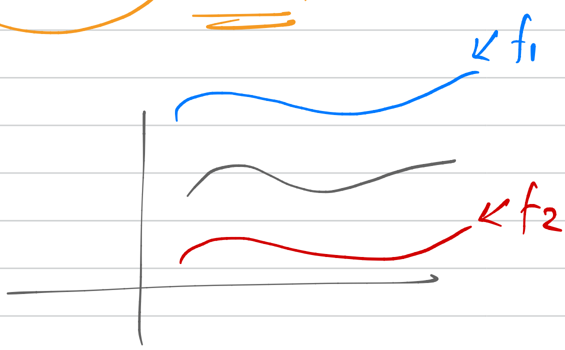
- for any x, $f(x)$
- $f(x) \geq 0$, $\quad f(x) \leq 1$
- $\int_a^b f(x)\,dx = 1$

$g : [a, b] \to [0, \infty)$

$$f(x) = \frac{g(x)}{\int_a^b g(y)\,dy} \Rightarrow \int_a^b f(x)\,dx = 1$$

scalar

function g



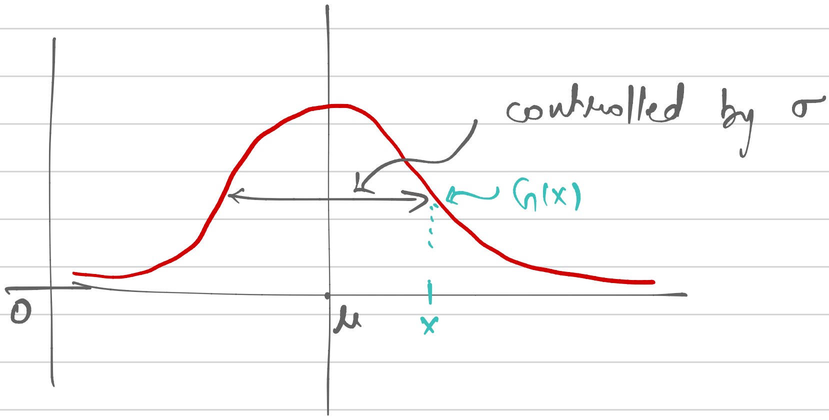
function $\frac{g}{0.1} = f_1$

function $\frac{g}{10} = f_2$

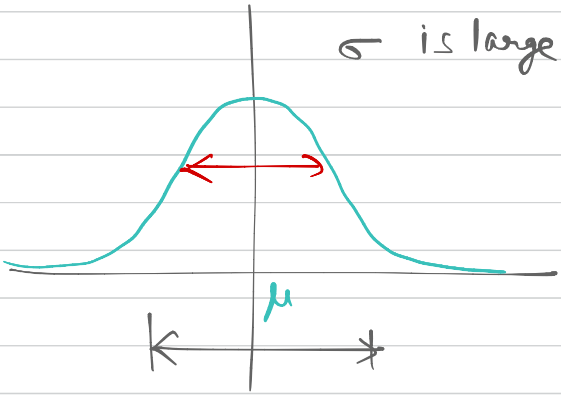
Gaussian distribution function (Gaussian)

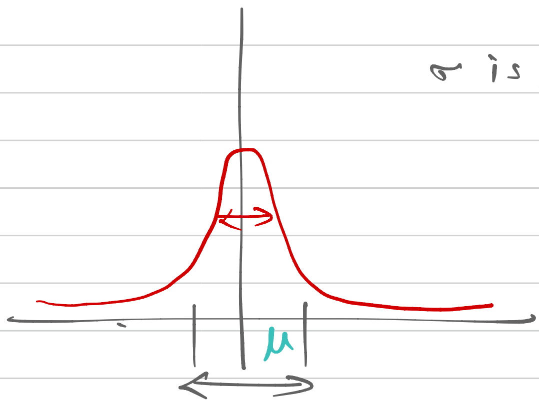
$$G(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$

$$\left.\begin{aligned} \sigma &= \text{standard deviation} \\ \mu &= \text{mean} \end{aligned}\right\} \text{parameters of Gaussian}$$



at $x = \mu$, $G(\mu)$ is maximum



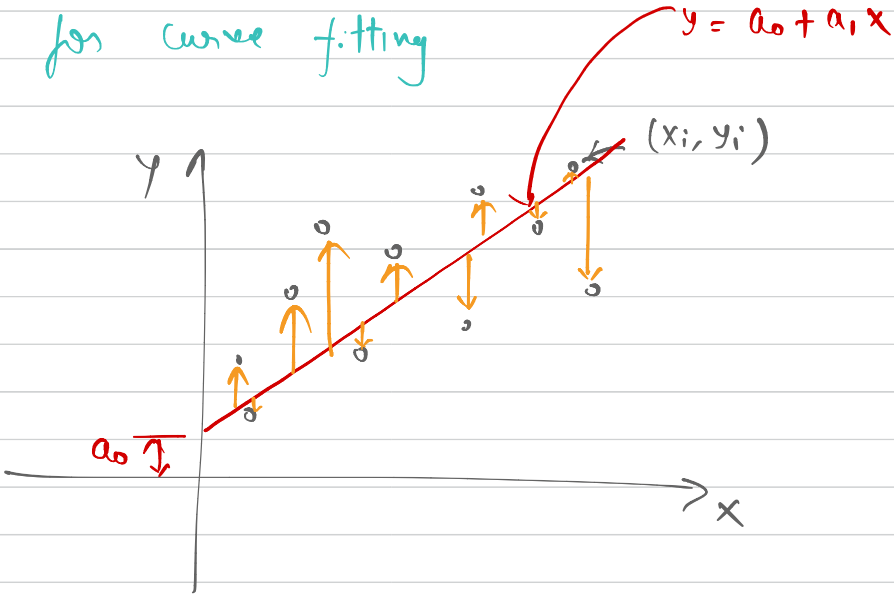


- Linear Regression for curve fitting

(x_i, y_i), $i = 1, 2, \ldots, n$

$y = y(x) = a_0 + a_1 x$

unknowns : a_0, a_1



Examples

(i) $y = a_0 + a_1 x$	linear curve	linear regression
(ii) $y = a_0 + a_1 x + a_2 x^2$	quadratic curve	linear regression
(iii) $y = a_0 + \sin(a_1 x) + \cos(a_2 x^2)^2$	nonlinear curve	nonlinear regression

$\hat{y} = \hat{y}(x) = a_0 + a_1 x$, $\hat{y}_i = \hat{y}(x_i)$

(x_1, y_1), (x_2, y_2), $\ldots$, (x_n, y_n)

$(x_1, \hat{y}_1)$, $(x_2, \hat{y}_2)$, $\ldots$, $(x_n, \hat{y}_n)$

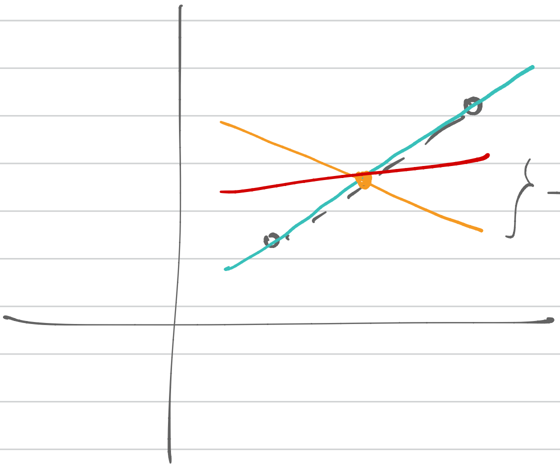
one possibility: $y_1 - \hat{y}_1$, $y_2 - \hat{y}_2$, $\ldots$, $y_n - \hat{y}_n$ $\qquad E_1 = \sum_{i=1}^{n} (y_i - \hat{y}_i)$

another possibility: $|y_1 - \hat{y}_1|$, $|y_2 - \hat{y}_2|$, $\ldots$, $|y_n - \hat{y}_n|$ $\qquad E_2 = \sum_{i=1}^{n} |y_i - \hat{y}_i|$

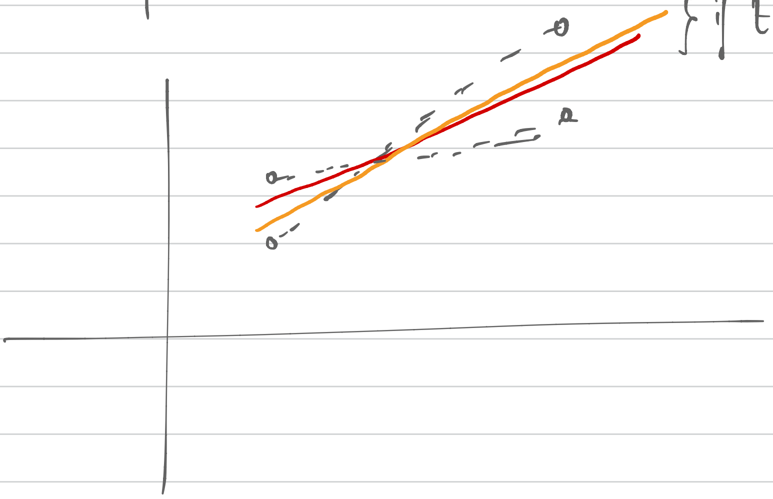
another possibility: $|y_1 - \hat{y}_1|^2$, $|y_2 - \hat{y}_2|^2$, $\ldots$, $|y_n - \hat{y}_n|^2$ $\qquad E_3 = \sum_{i=1}^{n} \underbrace{|y_i - \hat{y}_i|^2}_{\text{"square"}}$

least square method ← "find a_0, a_1 such that E_3 is minimum"

(x_1, y_1), (x_2, y_2)



$\}\rightarrow$ if E_1 is a total error



$\{$ if E_2 is a total error

$$E(a_0,a_1) = E = \sum_{i=1}^{n} \left| y_i - \hat{y}_i \right|^2 , \qquad \hat{y}_i = a_0 + a_1 X_i$$

"find a_0, a_1 such that $E(a_0, a_1)$ is minimum"

$$\rightarrow \boxed{\begin{aligned} \frac{\partial E}{\partial a_0} &= 0 \\ \frac{\partial E}{\partial a_1} &= 0 \end{aligned}}$$

$$\frac{\partial E}{\partial a_0} = \frac{\partial}{\partial a_0} \sum_{i=1}^{n}$$