Lecture 15

$$\textcircled{1}\quad a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$\textcircled{2}\quad a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$\textcircled{3}\quad a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$\Downarrow$$

$$\textcircled{1}\quad a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$\textcircled{2}\quad a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$\textcircled{3}\quad a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

$$\begin{bmatrix} a_{21} & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_2 \\ b_1 \\ b_3 \end{bmatrix}$$

$$\Downarrow$$

$$x_1 \longrightarrow x_2, \qquad x_2 \longrightarrow x_1$$

$$\textcircled{1}\quad a_{22}x_1 + a_{21}x_2 + a_{23}x_3 = b_2$$
$$\textcircled{2}\quad a_{12}x_1 + a_{11}x_2 + a_{13}x_2 = b_1$$
$$\textcircled{3}\quad a_{32}x_1 + a_{31}x_2 + a_{33}x_3 = b_3$$

$$\begin{bmatrix} a_{22} & a_{21} & a_{23} \\ a_{12} & a_{11} & a_{13} \\ a_{32} & a_{31} & a_{33} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$Ax = b$$

<u>Cramer's Rule</u> $\longrightarrow x$

<u>Gauss-Elimination</u> $\longrightarrow y$

$$Ax = b \qquad\qquad Ay = b$$

$$\Rightarrow \quad \boxed{A(x - y) = 0}$$

$\Rightarrow$ If there is a inverse of A, A^{-1}, then

$$A^{-1}(A(x-y)) = A^{-1}0$$

$$\Rightarrow A^{-1}A(x-y) = 0$$

$$\Rightarrow (x-y) = 0$$

$$\Rightarrow \boxed{x = y}$$

A^{-1} is a inverse of A then for any column vector x :

$$A^{-1}Ax = AA^{-1}x = x$$

$\Rightarrow$ For A^{-1} to exist, A must be such that $\det(A) \neq 0$

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

$$a_{11} = 0$$

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

$$a_{22} - a_{12}\frac{a_{21}}{a_{11}} = 0$$

1 ——

2 ——

⋮

j ——

j+1 ——

⋮

m ——

$$\begin{array}{ll} a_{jj} & x_j \\ a_{(j+1)j} & x_j \\ a_{mj} & x_j \end{array}$$

$$\begin{array}{ll} a_{(j+1)j} & x_j \\ a_{(j+2)j} & x_j \\ \vdots & \\ a_{mj} & x_j \end{array}$$