Assignment 5

Remark 1. The first 15 questions carry equal weights and are total worth 40 marks.
Remark 2. This assignment is only for those who need to improve their grades.

1. Name the primary sources of error in numerical methods. Describe them briefly.
2. What is the use of Gaussian elimination?
3. Name the two basic steps of Gaussian elimination. Shortly describe what they do.
4. Characterize the Jacobi and the Gauss-Seidel method in contrast to Gaussian elimination.
5. What is the main difference between the Jacobi and the Gauss-Seidel method?
6. How do you solve a linear system given in terms of a coefficient matrix A and a right-hand side vector b using `MATLAB` with one line of code?
7. What is an Eigenvalue problem?
8. For roots problem, what is the rate of convergence of Newton's method and fixed-point iteration method?
9. Write down Newton's method for finding the root of a function $f(x)$.
10. Explain with a short sketch what happens during one iteration of Newton's method.
11. Name the two differences between Gauss quadrature and piecewise polynormial approximation (e.g., Trapezoidal and Simpson's 1/3rd rules) of integration.
12. You are given a n-point Gauss quadrature rule to integrate a polynomial function $f(x)$. Up to what degree p of polynomials does the Gauss quadrature method yield the exact integral?
13. Which one of the two, Simpson's 1/3rd and 3/8th, require number of data points to be odd?
14. Use a Taylor series expansion at a given point x_i to predict the function value at a point $x_{i+1} = x_i + h$. Truncate the series after the third derivative term.
15. For the above Taylor series expansion, what is the order of the truncation error in terms of h?
16. **Numerical integration (20 marks).** Let

$$I = \int_{-2}^{4} (1 - x - 4x^3 + 2x^5)\,\mathrm{d}x.$$

Approximate I using

(a) single application of trapezoidal rule,
(b) composite application of trapezoidal rule with $n = 2$ and, separately, $n = 4$ subintervals,
(c) two-point Gauss quadrature rule.

17. **Numerical differentiation (20 marks).** A jet fighter's position on an aircraft carrier's runway was timed during landing:

t, s	0.0	0.5	1.0	2.0	3.0
x, m	150	158	200	225	262

where x is the distance from the end of the carrier.

(a) ***Estimate*** the velocity and acceleration of the aircraft at $t = 2\,\mathrm{s}$ using the first and second order centered finite difference formulas, respectively.

(b) ***Estimate*** the velocity and acceleration of the aircraft at $t = 1\,\mathrm{s}$ using a quadratic Lagrange polynomial fitted to three adjacent points at $t = 0.5$, 1.0 and $2.0\,\mathrm{s}$.

18. **Solving IVP-ODE (20 marks).** Consider the initial value problem

$$y'(t) = y(t) + 4t \tag{1}$$
$$y(t = 0) = 0. \tag{2}$$

We want to find n discrete solutions y at uniformly-spaced points in time, t_i, in the time interval $[0, 1]$.

(a) ***Provide*** the forward Euler approximation for (1),

(b) ***Implement*** forward Euler approximation and solve the problem using time step $\Delta t = 0.01$ and $\Delta t = 0.001$. ***Plot*** the approximate function y for both time steps.