Lecture 6

$$\boxed{\frac{dv}{dt} = g - \frac{C_d}{m} v^2}, \quad 0 < t \leq t_F$$

$$v(0) = 0$$

$$(*) \quad \frac{dv}{dt}(t) \approx \frac{v(t+h) - v(t)}{h}$$ provided h is small

$\rightarrow$ $t_0 = 0$, $t_1 = \Delta t$, $t_2 = 2\Delta t$, ..., $t_F = n\,\Delta t$

Δt : time step

$$v_i = v(t_i)$$

from $(*)$

$$\boxed{\frac{dv}{dt}(t_i) \approx \frac{v(t_i + \Delta t) - v(t_i)}{\Delta t}} \quad \text{—} \; (**)$$

from equation of v:

$$\frac{dv}{dt}(t_1) = g - C_d\, v(t_1)^2$$

$$\frac{dv}{dt}(t_2) = g - C_d\, v(t_2)^2$$

$$\vdots$$

$$\frac{dv}{dt}(t_{n-1}) = g - C_d\, v(t_{n-1})^2$$

general i, $\quad i = 1, 2, \ldots, n-1$ $\qquad v_i := v(t_i)$

$$\frac{dv}{dt}(t_i) \approx \frac{v(\underbrace{t_i + \Delta t}_{t_{i+1}}) - v(t_i)}{\Delta t} = g - \frac{C_d}{m} v(t_i)^2$$

$$\Rightarrow \frac{v_{i+1} - v_i}{\Delta t} = g - \frac{C_d}{m} v_i^2$$

$$\Rightarrow \boxed{v_{i+1} = v_i + \Delta t \left(g - \frac{C_d}{m} v_i^2\right)}$$

✓ v_0

✓ $v_1 = v_0 + \Delta t \left(g - \frac{C_d}{m} v_0^2\right)$

✓ $v_2 = v_1 + \Delta t \left(g - \frac{C_d}{m} v_1^2\right)$

$\vdots$

✓ $v_n = v_{n-1} + \Delta t \left(g - \frac{C_d}{m} v_{n-1}^2\right)$

$v(t_1) = v_0$

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for i = 2 : n
    v(i) = v(i-1)
        + Δt (g - Cd/m
        v(i-1)^2 )
end
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Errors in Numerical simulation

✗ (i) finite capacity of computer in representing numbers

✓ (ii) discretization error, truncation error, numerical error

✓, ✗ (iii) modeling error

E. P. Box ⟹ " All models are wrong, but some are useful "

✗ (iv) uncertainty / observation error / experimental error in data

✓ (v) wrong numerical implementation

Precision : a_i , $i = 1, \ldots, n$

$$|a_2 - a_1| > |a_3 - a_2| > \cdots > |a_n - a_{n-1}|$$

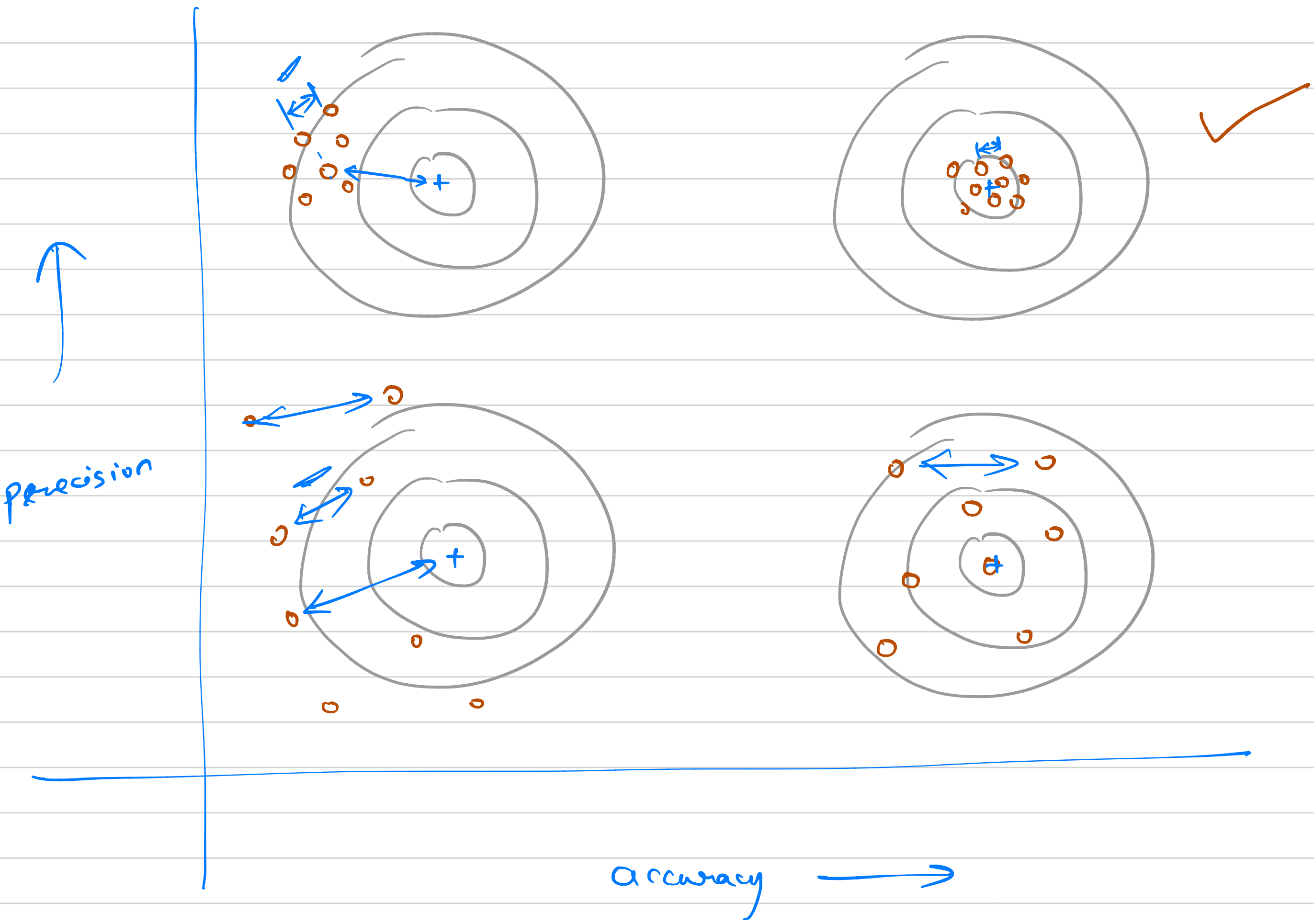
Accuracy : if suppose, I know true value a,

then

$$a - a_i$$

$$|a - a_1| > |a - a_2| > \cdots > |a - a_n|$$

$$< \qquad < \cdots <$$



Definition of errors :

True error : Applicable only if you know the true value

$$E_t = (\text{True value} - \text{Approximate value})$$

$V_{true} = 10\ m/s$ $\quad\quad$ $V_{true} = 10000\ m/s$

$V_{app} = 9\ m/s$ $\quad\quad$ $V_{approx} = 9999\ m/s$

$E_t = 1\ m/s$ $\quad\quad$ $E_t = 1\ m/s$

$$\boxed{e_t = \frac{E_t}{\text{True value}} \times 100\ \%}$$

$e_t = 10\%$ $\qquad$ $e_t = 0.01\ \%$

$$E_a = \frac{\text{Present approximate value} - \text{Previous approximate value}}{\text{Present approx. val.}} \times 100\%$$

$N(t_F; \Delta t)$ $\quad$ $N(t_F; \Delta t/2)$ $\quad$ $N(t_F; \Delta t/4)$ $\quad$ $\cdots$ $N(t_F; \Delta t/2^n)$

$$\frac{N(t_F;\Delta t/2) - N(t_F;\Delta t)}{N(t_F;\Delta t/2)} > \frac{N(t_F;\Delta t/4) - N(t_F;\Delta t/2)}{N(t_F;\Delta t/4)} > \cdots > \frac{N(t_F;\Delta t/2^n) - N(t_F;\Delta t/2^{n-1})}{N(t_F;\Delta t/2^n)}$$

$x \in [x_1, x_2]$

$f(y),\ f'(y),\ f''(y),\ \ldots$

$$f(x) = f(y) + f'(y)(y-x) + \frac{1}{2!} f''(y)(y-x)^2 + \frac{1}{3!} f'''(z)(y-x)^3$$

$z \in [x_1, x_2]$

$$f'(y) = \frac{df}{dy}, \quad \dot{f}(y) = \frac{df}{dy}, \quad f''(y) = \frac{d}{dy} f'(y) = \frac{d^2 f}{dy^2}(y)$$

$$f(x) = f(y) + \frac{f^{(1)}(y)}{1!}(y-x) + \frac{f^{(2)}(y)}{2!}(y-x)^2$$

$$+ \cdots + \frac{f^{(n)}(y)}{n!}(y-x)^n + \cdots$$