Lecture 2

Conservation laws

- Conservation of mass
- Conservation of linear momentum (Newton's 2nd law)
- Conservation of angular momentum
- Conservation of energy
- second law of thermodynamics

$\eta = \eta(t)$

$$\frac{d\eta}{dt} \geq 0$$

Constitutive law

δ = change in length of spring

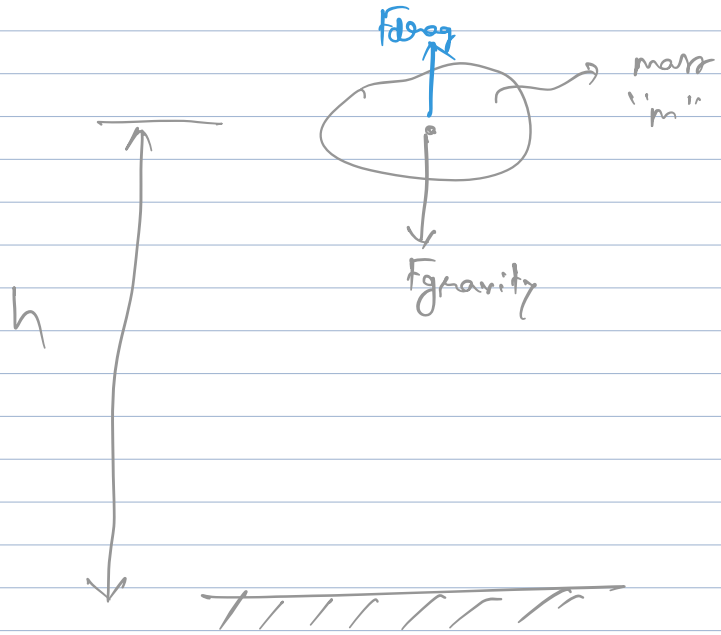
How do I know the force in the spring

force is a function of change in length

f = force, δ = change in length

$$f = f(\delta) = -k\delta$$

Example 1 :



Conservation of linear momentum

Rate of change of linear momentum

$$= F_{internal} + F_{external}$$

$F_{internal}$ → 0 (Rigid body assumption)

$F_{external}$ → $F_{gravity}$, F_{drag}

Linear momentum = mv

$F_{internal} = 0$

$F_{gravity} = mg$

$F_{drag} =$

↓ +

Observations

- F_{drag} is opposite to v
- F_{drag} is proportional to $|v|$

$F_{drag} \propto \Theta |v|(-v)$

$= C_d |v| (-v)$

$$F_{drag} = -C_d |v| v$$

- F_{drag} depends on environmental conditions
- shape of an object
- mass of an object

$$\frac{d}{dt}(mv) = mg - C_d |v| v$$

$$\Rightarrow \left(\frac{dm}{dt}\right)^{0} v + m\frac{dv}{dt} = mg - C_d |v| v$$

$v \geq 0$

$|v| = v$

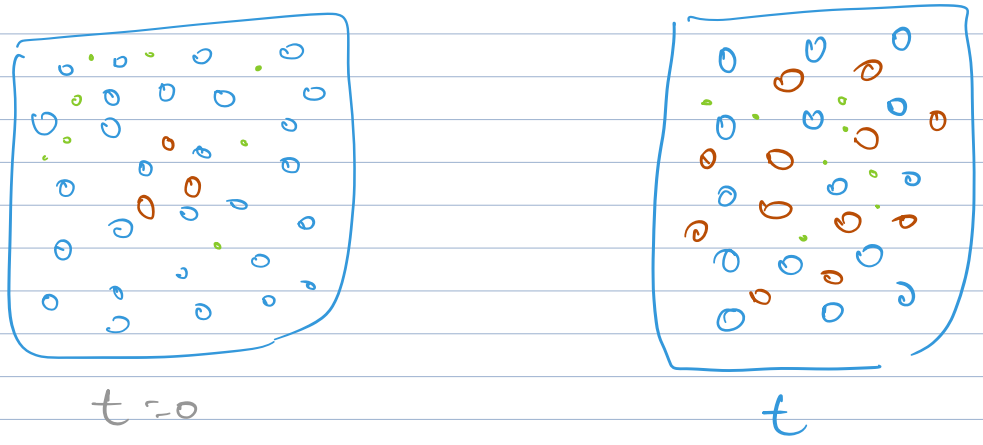
$$\Rightarrow m\frac{dv}{dt} = mg - C_d v^2$$

$$\Rightarrow \boxed{\frac{dv}{dt} = g - \frac{C_d}{m} v^2}$$

Initial condition $v(t=0) = 0$

Example 2 :

$N = N(t)$ = number of cancerous cells at time t



$t = 0$ $\qquad$ t

$N(0)$ $\qquad$ $N(t)$

Conservation of mass

rate of change of cancerous cells

= Cancerous cell growth rate − kill effect due to drugs

"α" N "α" $\log\left(\frac{1}{N}\right)$

α N α $\delta(t)$ ← amount of drug at time t

$$\frac{d}{dt}(mN) = \underbrace{\gamma N \log\left(\frac{1}{N}\right)}_{\text{growth}} - C\delta(t)N$$

γ = growth rate parameter

C = kill-effect parameter

$\delta = \delta(t)$ = drug amount at time t

m = mass of cancerous cell

3) $$\frac{dN}{dt} = \left(\frac{\gamma}{m}\right) N \log\left(\frac{1}{N}\right) - \left(\frac{c}{m}\right) \delta N$$

$$N(0) = N_0$$