Lecture 16

$$a_{11} x_1 + a_{12} x_2 + a_{13} x_3 = b_1$$

$$a_{21} x_1 + a_{22} x_2 + a_{23} x_3 = b_2$$

$$a_{31} x_1 + a_{32} x_2 + a_{33} x_3 = b_3$$

$$\left[\begin{array}{ccc:c} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{array} \right]$$

$\Downarrow$

$$f_{21} = \frac{a_{21}}{a_{11}}$$

$$f_{31} = \frac{a_{31}}{a_{11}}$$

$$\left[\begin{array}{ccc:c} a_{11} & a_{12} & a_{13} & b_1 \\ 0 & a_{22}^{(1)} & a_{23}^{(1)} & b_2^{(1)} \\ 0 & a_{32}^{(1)} & a_{33}^{(1)} & b_3^{(1)} \end{array} \right]$$

$\underline{a_{11} \quad \text{is a pivot}}$

$\Downarrow$

$$f_{32} = \frac{a_{32}^{(1)}}{a_{22}^{(1)}}$$

$$\left[\begin{array}{ccc:c} a_{11} & a_{12} & a_{13} & b_1 \\ 0 & a_{22}^{(1)} & a_{23}^{(1)} & b_2^{(1)} \\ 0 & 0 & a_{33}^{(2)} & b_3^{(2)} \end{array} \right]$$

$\underline{a_{22}^{(1)} \text{ is a pivot}}$

$\underline{a_{33}^{(2)} \text{ is a pivot}}$

Transpose of matrix

$$
A_{m \times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \Rightarrow \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & & a_{m2} \\ \vdots & \vdots & & \vdots \\ a_{1n} & a_{2n} & & a_{mn} \end{bmatrix} B_{n \times m}
$$

A^{-1}

$$
B_{n \times m} = [b_{ij}] = [a_{ji}] = A^T_{m \times n}
$$

$$
b_{12} = a_{21}
$$

$$
b_{n2} = a_{2n}
$$

Types of matrices

- **Square matrix** $\qquad A_{n \times m}, \qquad n = m$

- **Symmetric matrix** $\qquad$ A square matrix $A_{n \times n}$ is

  symmetric if

  $$
  A^T = A \qquad \Rightarrow \qquad a_{ji} = a_{ij}
  $$
  for all $i, j = 1, 2, \dots n$

$$
\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \Rightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}
$$

$T\Theta$ operation

- **Diagonal matrix**

$$\begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix}$$

$\Rightarrow a_{ij} = 0$ for all $i \neq j$

- **Upper triangle matrix**

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}$$

$\Rightarrow a_{ij} = 0$ for all $i > j$

- **Lower triangle matrix**

$$\begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$a_{ij} = 0$ for all $i < j$