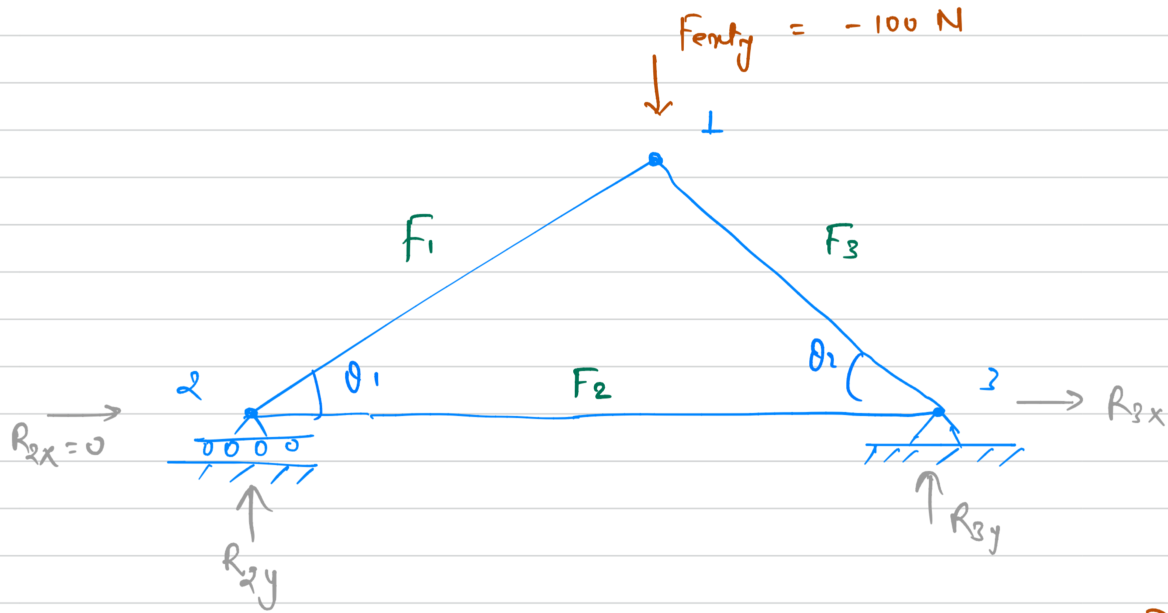
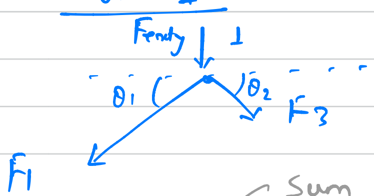
Lecture 11

System of linear equations



Truss structure

Point 1



sum of horizontal forces = 0

sum of vertical forces = 0

(1) $-F_1 \cos\theta_1 + F_3 \cos\theta_2 = 0$

(2) $-F_1 \sin\theta_1 - F_3 \sin\theta_2 + F_{enty} = 0$

Don't know

— F_1, F_2, F_3

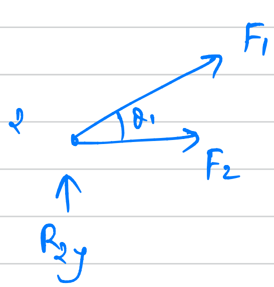
— R_{2y}, R_{3x}, R_{3y}

what do we know

(i) $F_{enty} = -100N$

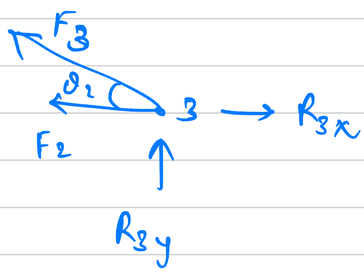
(ii) θ_1, θ_2 are known

(iii) Truss structure is at rest



(3) $F_1 \cos\theta_1 + F_2 = 0$

(4) $F_1 \sin\theta_1 + R_{2y} = 0$



(5) $-F_3 \cos(\theta_2) - F_2 + R_{3x} = 0$

(6) $F_3 \sin(\theta_2) + R_{3y} = 0$

→ Got 6 system of equations

↓

equations are linear in unknowns $F_1, F_2, F_3, R_{2y}, R_{3x}, R_{3y}$

$F_1 F_2,\ F_2^2,\ \ldots,\ R_{3x} R_{3y}$

$\exp(-F_1)$

$F_1, \quad F_2, \quad F_3, \quad R_{2y}, \quad R_{3x}, \quad R_{3y}$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6$

(1) — $a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + a_{14} x_4 + a_{15} x_5 + a_{16} x_6 = b_1$

$a_{11} = -\cos\theta_1$, $a_{12} = 0$, $a_{13} = \cos(\theta_2)$, $a_{14} = 0$, $a_{15} = 0$, $a_{16} = 0$

$b_1 = 0$

(2) — $a_{21} x_1 + a_{22} x_2 + \cdots + a_{26} x_6 = b_2$

$a_{21} = -\sin(\theta_1)$, $a_{22} = 0$, $a_{23} = -\sin(\theta_2)$, $a_{24} = a_{25} = a_{26} = 0$,

$b_2 = -$ Fenty

(3) — –

(6) $a_{61}x_1 + a_{62}x_2 + \cdots + a_{66}x_6 = b_6$

$a_{61} = 0 = a_{62} = a_{64} = a_{65}$, $a_{63} = \sin(\theta_2)$, $a_{66} = 1$, $b_6 = 0$

$$\boxed{\begin{aligned} a_{11}x_1 + \cdots + a_{16}x_2 &= b_1 \\ &\vdots \\ a_{61}x_1 + \cdots + a_{66}x_6 &= b_6 \end{aligned}}$$

- $\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_6 \end{bmatrix}$ is unknown
- $\begin{bmatrix} b_1 \\ \vdots \\ b_6 \end{bmatrix}$ is known
- $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{16} \\ a_{21} & a_{22} & \cdots & a_{26} \\ \vdots \\ a_{61} & a_{62} & \cdots & a_{66} \end{bmatrix}$ is also known

Column vector: small letters to define a column vector

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$ $\Rightarrow$ This is a column vector with n elements

row vector:

$(x) = [x_1, x_2 \ldots, x_n]$ $\Rightarrow$ This is a row vector with n elements.

- Matrix notation:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2n} \\ \vdots & & & & & \\ a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{in} \\ \vdots & & & & & \\ a_{m1} & a_{m2} & \cdots & a_{mj} & \cdots & a_{mn} \end{bmatrix}$$

$m \times n$ matrix

# of rows   # of columns

element of A: the row number

the column number

Example: a_{2n} = element sitting at 2^{nd} row and n^{th} column

a_{ij} = element sitting at i^{th} row and j^{th} column

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• Algebra on matrix

• addition

$A_{m\times n}$, $B_{l\times r}$ $\Rightarrow$ addition is defined only if

$m = l$
$n = r$

$$C_{m\times n} = A_{m\times n} + B_{l\times r}$$

$l \times r$

$$= \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} & - - & a_{1n}+b_{1n} \\ \vdots & & & \\ \vdots & & a_{ij}+b_{ij} & \\ & & & a_{mn}+b_{mn} \end{bmatrix}$$

$$C = A + B \Rightarrow [c_{ij}] = [a_{ij} + b_{ij}]$$

element at 1st row and 3rd column

$$C_{13} = a_{13} + b_{13}$$

• Multiplication by scalar (number)

$A \longrightarrow$ of any size $m \times n$

$$B = \alpha A = \begin{bmatrix} \alpha a_{11} & \alpha a_{12} & - - & \alpha a_{1n} \\ & & \alpha a_{ij} & \\ & & & \alpha a_{mn} \end{bmatrix}$$

number

$\Rightarrow \quad [b_{ij}] = [\alpha a_{ij}]$

- Subtraction $\quad A_{m\times n}, \quad B_{m\times n}$

$$C_{m\times n} = A_{m\times n} - B_{m\times n} = A_{m\times n} + (-1)\, B_{m\times n}$$

- multiplication of matrices

$$\underbrace{\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}_{3\times 2}}_{A} \times \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \end{bmatrix}_{2\times 3}}_{B} = C$$

$$\boxed{\begin{aligned} &[a_1 \ a_2 \ \cdots \ a_n] \times \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \\ &= a_1 b_1 + a_2 b_2 + \cdots + a_n b_n \\ &\Downarrow \\ &\text{size of row vector} = \text{size of column vector} \end{aligned}}$$

$$C_{11} = [1, 2] \times \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1 + 4 = 5$$

$$C_{12} = [1, 2] \times \begin{bmatrix} 1 \\ 3 \end{bmatrix} = 7$$

$$C_{21} = [3, 4] \times \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 11$$

C_{22}, $\quad C_{31}$, $\quad C_{32}$

$$C = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1q} \\ b_{21} & b_{22} & \cdots & b_{2q} \\ \vdots & & & \\ b_{l1} & b_{l2} & \cdots & b_{lq} \end{bmatrix}$$

c_{ij} = (ith row of A) × (jth column of B)

$$= [a_{i1}, a_{i2}, \ldots, a_{in}] \times \begin{bmatrix} b_{1j} \\ b_{2j} \\ \vdots \\ b_{lj} \end{bmatrix} = a_{i1} b_{1j} + a_{i2} b_{2j} + \cdots + a_{in} b_{lj}$$

$$= \sum_{k=1}^{n} a_{ik} b_{kj}$$

size of $C = m \times q$

$$\boxed{\begin{aligned} C &= A \times B \\ [c_{ij}] &= \left[\sum_{k=1}^{n} a_{ik} b_{kj} \right] \end{aligned}}$$

$$c_{12} = \sum_{k=1}^{n} a_{1k} b_{k2} = a_{11} b_{12} + a_{12} b_{22} + \cdots + a_{1n} b_{n2}$$

defined only if $n = l$

or in other words

$A \times B$ is defined only if (# columns of A) = (# rows of B)

$D = B_{l \times q} \times A_{m \times n} \Rightarrow$ # columns of B = # rows of A

$\Downarrow$

$[d_{ij}] = \left[\sum_{k=1}^{m} b_{ik} a_{kj} \right] \Rightarrow$ size of $D = l \times n$