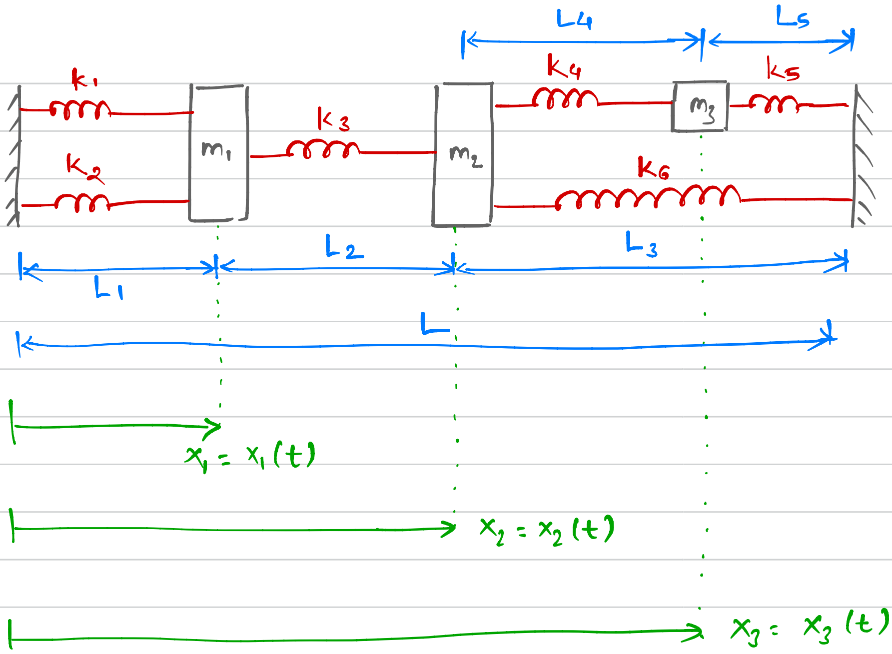


- $x_1 = x_1(t)$ = coordinate of m_1 from left wall

  $x_2 = x_2(t)$ = coordinate of m_2 from left wall

  $x_3 = x_3(t)$ = coordinate of m_3 from left wall

- L_1, L_2, L_3, L_4, L_5 = Original lengths of springs (Given)

  Clearly
  - $L_4 + L_5 = L_3$
  - $L_1 + L_2 + L_3 = L$

- $k_1, k_2, k_3, k_4, k_5, k_6$ = Spring stiffness (Given)

- m_1, m_2, m_3 = masses (Given)

- Displacement of masses

$$y_1 = x_1 - L_1$$

$$y_2 = x_2 - (L_1 + L_2)$$

$$y_3 = x_3 - (L_1 + L_2 + L_4)$$

since $x_1(0) = L_1$, $x_2(0) = L_1 + L_2$
$x_3(0) = L_1 + L_2 + L_4$

y_1, y_2, y_3 are displacements of m_1, m_2, m_3 from original position of these masses.

Problem 1: Let

- $k_1 = k_2 = k_4 = k_5 = 1$, $k_3 = k_6 = 2$.
- $m_3 = 1$, $m_1 = m_2 = 2$

Find characteristic solution for y_1, y_2, y_3.

Problem 2: Consider initial conditions

$$y_1(0) = y_2(0) = y_3(0) = 0$$

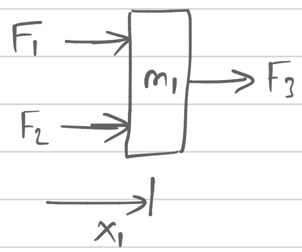
$$\frac{dy_1}{dt}(0) = a, \quad \frac{dy_2}{dt}(0) = b, \quad \frac{dy_3}{dt}(0) = c$$

Fix $a = 1$, $b = 2$, $c = -1$.

Find complete solution y_1, y_2, y_3 that satisfies the initial condition.

- Derivation of equations for y_1, y_2, y_3.

(A) m_1



δ_1 = change in length of spring 1

$= x_1 - L_1 = y_1$

δ_2 = change in length of spring 2

$= x_1 - L_1 = y_1$

δ_3 = change in length of spring 3

$= x_2 - x_1 - L_2$

$$= y_2 + \cancel{L_1} + \cancel{L_2} - (y_1 + \cancel{L_1}) - \cancel{L_2}$$

$$= y_2 - y_1$$

force

$$F_1 = -k_1 \delta_1 = -k_1 y_1$$

$$F_2 = -k_2 \delta_1 = -k_2 y_1$$

$$F_3 = k_3 \delta_3 = k_3 (y_2 - y_1)$$

if $\delta_3 > 0$ spring 3 will pull i.e. F_3 will be +ve

If $\delta_3 < 0$ spring 3 will push i.e. F_3 will be -ve

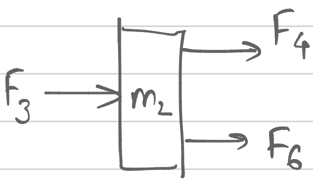
Thus from conservation of linear momentum

$$m_1 \frac{d^2 x_1}{dt^2} = -k_1 y_1 - k_2 y_2 + k_3 (y_2 - y_1)$$

$\because$ $y_1 = x_1 - L_1 \Rightarrow \frac{dy_1}{dt} = \frac{dx_1}{dt}, \quad \frac{d^2 y_1}{dt^2} = \frac{d^2 x_1}{dt^2}$

$$\Rightarrow \boxed{m_1 \frac{d^2 y_1}{dt^2} = -(k_1 + k_2 + k_3) y_1 - k_3 y_2}$$

(B) m_2



$$\delta_3 = x_2 - x_1 - L_2 = y_2 - y_1$$

$$\delta_4 = x_3 - x_2 - L_4$$

$$= y_3 + L_1 + L_2 + L_4 - (y_2 + L_1 + L_2) - L_4$$

$$\Rightarrow \delta_4 = y_3 - y_2$$

$$\delta_6 = (L - x_2) - L_3$$

$$= L_1 + L_2 + L_3 - (y_2 + L_1 + L_2) - L_3$$

$$\Rightarrow \delta_6 = -y_2$$

$F_3 \rightarrow$ if $\delta_3 > 0$, $F_3 < 0$

if $\delta_3 < 0$, $F_3 > 0$

$$F_3 = -K_3 \delta_3 = -k_3 (y_2 - y_1)$$

$F_4 \rightarrow$ if $\delta_4 > 0$, $F_4 > 0$

if $\delta_4 < 0$, $F_4 < 0$

$$F_4 = k_4 \delta_4 = k_4 (y_3 - y_2)$$

$F_6 \rightarrow$ if $\delta_6 > 0$, $F_6 > 0$

if $\delta_6 < 0$, $F_6 < 0$

$$F_6 = k_6 \delta_6 = -k_6 y_2$$

So

$$m_2 \frac{d^2 x_2}{dt^2} = -k_3 (y_2 - y_1) + k_4 (y_3 - y_2) - k_6 y_2$$

$$\Downarrow$$

$$\frac{d^2 y_2}{dt^2}$$

$$\boxed{m_2 \frac{d^2 y_2}{dt^2} = k_3 y_1 + (-k_3 - k_4 - k_6) y_2 + k_4 y_3}$$

(c) m_3

$$F_4 \longrightarrow \boxed{m_3} \longrightarrow F_5$$

$$\delta_4 = y_3 - y_2$$

$$\delta_5 = (L - x_3) - L_5$$

$$= L - (y_3 + L_1 + L_2 + L_4) - L_5$$

$$\Rightarrow \delta_5 = -y_3$$

$$F_4 \longrightarrow \left.\begin{array}{ll} \text{if } \delta_4 > 0, & F_4 < 0 \\ \text{if } \delta_4 < 0, & F_4 > 0 \end{array}\right\} \Rightarrow F_4 = -k_4 \delta_4 = -k_4 (y_3 - y_2)$$

$$F_5 \longrightarrow \left.\begin{array}{ll} \text{if } \delta_5 > 0, & F_5 > 0 \\ \text{if } \delta_5 < 0, & F_5 < 0 \end{array}\right\} \Rightarrow F_5 = k_5 \delta_5 = -k_5 y_3$$

So

$$m_3 \frac{d^2 x_3}{dt^2} = -k_4 (y_3 - y_2) - k_5 y_3$$

$$\Rightarrow \boxed{m_3 \frac{d^2 y_3}{dt^2} = k_4 y_2 + (-k_4 - k_5) y_3}$$

Combining

(I) System of 2nd order Linear ODEs

$$\frac{d^2 y_1}{dt^2} = \frac{-(k_1 + k_2 + k_3)}{m_1} y_1 + \frac{k_3}{m_1} y_2$$

$$\frac{d^2 y_2}{dt^2} = \frac{k_3}{m_2} y_1 - \frac{(k_3 + k_4 + k_6)}{m_2} y_2 + \frac{k_4}{m_2} y_3$$

$$\frac{d^2 y_3}{dt^2} = \frac{k_4}{m_3} y_2 - \frac{(k_4 + k_5)}{m_3} y_3$$

(II) Initial conditions

(1.) $y_1(0) = y_2(0) = y_3(0) = 0$

(2.) $\frac{dy_1}{dt}(0) = a$, $\frac{dy_2}{dt}(0) = b$, $\frac{dy_3}{dt}(0) = c$

Second problem set

Consider $u = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$, $A = \begin{bmatrix} 3 & 1 \\ -2 & 6 \end{bmatrix}$

$$\frac{du}{dt} = Au$$

1. find characteristics solution
2. Let a, b such that

$$u(0) = \begin{bmatrix} a \\ b \end{bmatrix}$$

find complete solution satisfying initial condition

$$u(0) = \begin{bmatrix} a \\ b \end{bmatrix}$$

3. Inverse problem Suppose $u(1) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

find a, b from solution of (2) such that

$$u(1) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

We are solving for initial condition of u based on observed value of u at $t=1$.