Assignment 3

Problem set 1 (60 marks)

Consider a spring-mass system in fig. 1. Let k_i, $i = 1, 2, ..., 6$, is the stiffness of spring i, L_1, L_2, L_3, L_4, L_5 are lengths of various springs at time $t = 0$. Suppose $x_i = x_i(t)$, $i = 1, 2, 3$, is the position of mass m_i from the left wall at time t.

Let $y_1 = x_1 - L_1$, $y_2 = x_2 - (L_1 + L_2)$, and $y_3 = x_3 - (L_1 + L_2 + L_4)$ are the displacements of the three masses at time t. Using the conservation of linear momentum principle, we arrive at the following linear system of second order ordinary differential equations:

$$m_1 \frac{\mathrm{d}^2 y_1}{\mathrm{d}t^2} = -(k_1 + k_2 + k_3) y_1 + k_3 y_2,$$
$$m_2 \frac{\mathrm{d}^2 y_2}{\mathrm{d}t^2} = k_3 y_1 - (k_3 + k_4 + k_6) y_2 + k_4 y_3,$$
$$m_3 \frac{\mathrm{d}^2 y_3}{\mathrm{d}t^2} = k_4 y_2 - (k_4 + k_5) y_3, \qquad (1)$$

To solve the above coupled system of equations, we require total 6 initial conditions. We take:

$$y_1(0) = y_2(0) = y_3(0) = 0, \qquad \frac{\mathrm{d}y_1}{\mathrm{d}t}(0) = a, \quad \frac{\mathrm{d}y_2}{\mathrm{d}t}(0) = b, \quad \frac{\mathrm{d}y_3}{\mathrm{d}t}(0) = c. \qquad (2)$$

Here, a, b, c are the three given numbers.

Remark 1. See the supplementary file 'A3_derivation_spring_system.pdf' for the derivation.

Parameters. Let $k_1 = k_2 = k_4 = k_5 = 1$, $k_3 = k_6 = 2$, $m_3 = 1$, $m_1 = m_2 = 2$. Also, $a = 1$, $b = 2$, and $c = -1$.

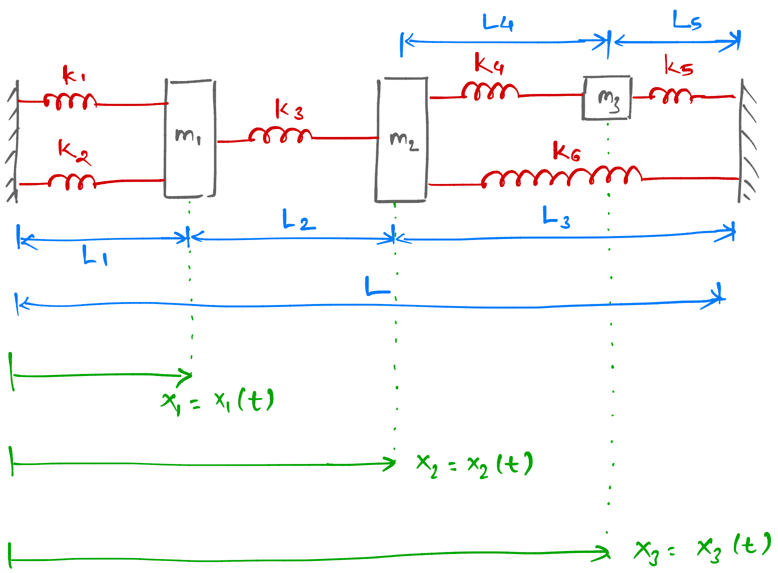

Figure 1: Spring-mass system.

Problem 1 (30 marks). ***Derive*** *characteristic (incomplete) solutions* of eq. (1). Also, plot the three characteristics solutions separately between time interval $[0, 10]$. These three solutions show the three characteristics modes of vibration of spring-mass system.

Problem 1 (30 marks). ***Find*** *the complete solution* that satisfies eq. (1) and eq. (2).

Problem set 2 (40 marks)

Let $u_1 = u_1(t)$ and $u_2 = u_2(t)$ satisfies the following system of first order ordinary differential equation:

$$\frac{\mathrm{d}u_1}{\mathrm{d}t} = 3u_1 + u_2,$$
$$\frac{\mathrm{d}u_2}{\mathrm{d}t} = -2u_1 + 6u_2. \tag{3}$$

Problem 1 (20 marks). ***Derive*** *characteristic (incomplete) solutions* of eq. (3).

Problem 2 (10 marks). Let u_1, u_2 also satisfies

$$u_1(0) = a, \qquad u_2(0) = b, \tag{4}$$

where a, b are two parameters.

Find *the complete solution* that satisfies eq. (4) for the two parameters a and b. Clearly, the solution u_1 and u_2 will include a and b.

Problem 3 (10 marks). Often times we deal with a situation where we observe system at specific time and want to know the state of system at past times. For example, we may want to know the value of a and b in eq. (4) based on what we know about the solution u_1 and u_2 at some later time T.

Using, $T = 10$,

$$u_1(10) = 1, \qquad u_2(10) = 2, \tag{5}$$

find the initial condition parameters a and b. Use the solution u_1 and u_2 you obtained in **Problem 2**.