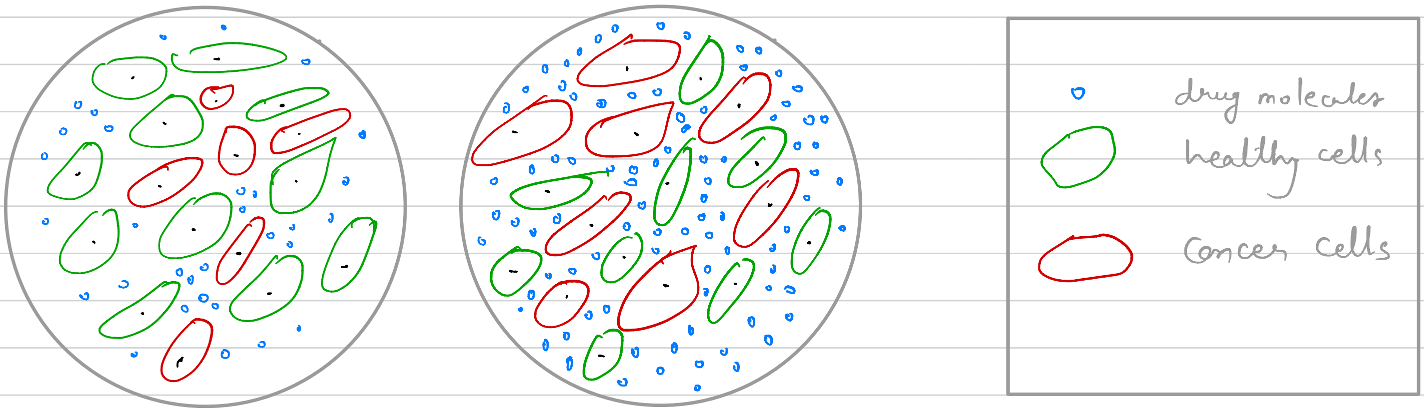
# Model of growth of tumor (ODE)




$t = 0$ $\qquad$ $t = t_F$

Consider a cell culture dish with initially $N_0$ number of cancer cells, some healthy cells, and some amount of drug molecules.

Our goal is to

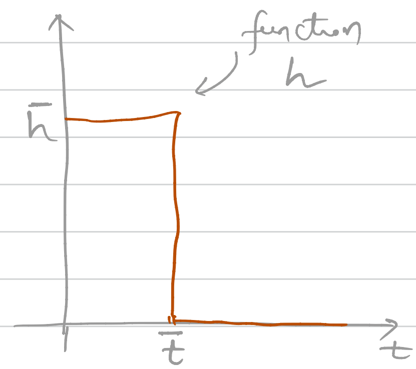
(i) develop equation that specifies number of cancer cells at any time $t$, $0 \le t \le t_F$

(ii) incorporate the effect of drug in the model of cancer cells

Let $h = h(t)$ is the function indicating the total drug in the dish. It is fixed given function.

We will assume that function $h$ is given by

$$h(t) = \begin{cases} \bar{h} & \text{for } 0 \le t < \bar{t} \\ 0 & \text{for } \bar{t} \le t \end{cases} \Rightarrow$$




where $\bar{h}$ = dose amount (given) (g)

$\bar{t}$ = time at which dose is stopped (given) (day)

Let us define

$N = N(t)$ = number of cancer cells at time $t$

$t$ = time in units of "days" ($0 \le t \le t_F$, $t_F$ is final time)

$m$ = mass of a single cell (we assume that all cells, be it cancer or healthy have same mass) (unit of gram "g")

**Growth of cancer cells** Cancer cells multiply at a faster rate but typically: (i) the rate of growth varies from cell to cell

(ii) the growth rate is not always same as the number of cells increase or decrease

We consider following model

$$\boxed{f_{growth}(t) = r\, m\, N(t) \ln\left(\frac{N_\infty}{N(t)}\right)}$$

natural log

where $N_\infty$ = the maximum number of cancer cells possible to have in a dish (given number) (also called carrying capacity)

$r$ = growth rate (units of $r$ is $\frac{1}{day}$)

$\ln\left(\frac{N_\infty}{N(t)}\right)$ ensures that growth of cells decay with increasing $N(t)$ so that the number of cancer cells do not exceed the limit $N_\infty$

**Effect of drug :** Drug kill cancer cells and we consider following simple model of amount of cells killed by drug (number of cells killed should be proportional to both the amount of drugs and number of cells that exist)

$f_{drug}(t)$ = amount of cells killed by drug at time $t$

$$\boxed{f_{drug}(t) = k\, h(t)\; m\, N(t)}$$

where $k$ = kill rate $\left(\frac{1}{g}\,\frac{1}{day}\right)$

$h(t)$ = amount of drug at time $t$ ($g$)

Applying conservation of mass :

rate of change of mass of cancer cells = rate of growth of cancer cells − rate at which cancer cells are killed

$$\frac{d}{dt}\left(m N(t)\right) = f_{growth}(t) - f_{drug}(t)$$

( assuming $m$ is constant )

$$\Rightarrow \quad \cancel{m} \frac{dN(t)}{dt} = r \cancel{m} N(t) \ln\left(\frac{N_\infty}{N(t)}\right) - k \cancel{m} h(t) N(t)$$

$$\Rightarrow \quad \boxed{\frac{dN(t)}{dt} = r N(t) \ln\left(\frac{N_\infty}{N(t)}\right) - k h(t) N(t)}$$

with $\boxed{N(0) = N_0}$ ( Initial condition )

In above

(1.) $r, k, N_\infty, N_0$ are given

(2.) $h(t) = \begin{cases} \bar{h} & \text{if } 0 \le t < \bar{t} \\ 0 & \text{otherwise} \end{cases}$

(3.) $\bar{h}, \bar{t}$ are given

(4.) final time $t_f$ is also given.