Assignment 1

Consider the following model of growth of cancer cells in a cell culture dish:

$$\frac{dN(t)}{dt} = rN(t)\ln\left(\frac{N_\infty}{N(t)}\right) - kh(t)N(t), \qquad \forall 0 < t \le t_F, \tag{1}$$

with initial condition

$$N(0) = N_0. \tag{2}$$

Here, $N = N(t)$ is the number of cancerous cells at time t, r the proliferation or growth rate (in units of 1/day), N_∞ a number specifying the maximum number of cells in a dish (carrying capacity), k a number indicating the rate at which drug kills cancer cells (in units of 1/day/g, 'g' for gram), and $h = h(t)$ the mass of drug molecules at time t (in units of g).

In (1), except function $N(t)$, all other parameters such as N_∞, N_0, r, k, t_F and the function $h(t)$ are given. As a function of $h(t)$, we take the following 'step' function:

$$h(t) = \begin{cases} \bar{h}, & \text{if } 0 \le t < \bar{t}, \\ 0.1\bar{h}, & \text{otherwise}, \end{cases} \tag{3}$$

where again the values of $\bar{h}$ and $\bar{t}$ are known.

Remark 1. The derivation of above function is in a file supplement to this assignment file.

Parameter values. Take $t_F = 20$ (days), $N_0 = 100$, $N_\infty = 10000$, $r = 0.7$ (1/day), $k = 100$ (1/day/g), $\bar{h} = 0.01$ (g), and $\bar{t} = 0.5t_F$ (days). Further, consider discrete times between 0 and t_F with spacing $\Delta t = 0.1$ (days).

Problem 1 (50 marks). Write down the numerical approximation of (1) (similar to the gravity problem worked out in the class) and compute $N(t_F)$ using the parameters specified above. Also, plot the values of $N(t_i)$ at discrete times t_i using MATLAB plot function.

Remark 2. If you take $\Delta t = 1$ (days), the number of cancer cells $N(t_F)$ at the final time is about 8670. This should help you in verifying your implementation.

Problem 2 (20 marks). Run your code with four different $\Delta t = 1, 0.1, 0.01, 0.001$ and list the value of $N(t_F)$ for each case.

Problem 3 (30 marks). Instead of 'step' function for h, try another function, say a function that linearly increases from 0 to $\bar{h}$ from time 0 to $\bar{t}$, and for time above $\bar{t}$, $h(t) = 0.1\bar{h}$. Using either the new function I just described or your own new function, compute $N(t_F)$ with parameters listed above and with $\Delta t = 0.1$. Compare with the result for 'step' function in (3). You could try any other function instead of a function described here.